# RADIATION OF AN ACCELERATED CHARGE

Enrique Cantera del Río

C/ Padre Benito Menni 6-2-E 47013 Valladolid,Spain

***ABSTRACT***

*Rethinking the dynamics of an accelerated charge from classical concepts. From the idea that radiation comes from kinetic energy and managing the problem of the auto-energy of a point charge, a system of non-linear dynamic equations are found and results amenable to experimental verification. In developing, a relationship between the principle of causality, which affects the direction of time, and the constancy of mass appears. Another consequence are the fluctuations in the motion of particles, compatible with Brownian motion and Heisenberg´s indeterminacy principle. The case of gravitational acceleration is also analyzed, concluding that no electromagnetic radiation is possible and there is no electric field that can produce a constant acceleration on a point charge. Thus the constant acceleration is an exclusive feature of gravity.*

***KEYWORDS***

*Radiation reaction force, Abraham-Lorentz equation, Causality.*

## 1. INTRODUCTION

The behaviour of an accelerated electric charge, regardless of the accelerating force, is a limit problem in classical physics. The radiation of a system of charges is described by *Poynting's theorem*, which is a logical consequence of Maxwell's equations. The key point is that the *Poynting vector*, which appears in this theorem, is interpreted as a flow of radiant energy using the principle of the conservation of energy in an electromagnetic system. From this perspective, it seems that radiation, similar to potential energy, is a behaviour that is associated with a system of charges and not with individual charges. In this sense, the literature discusses *dipole radiation*, *quadrupole radiation,* etc[4]. However, in classical theory, it is immediately clear that the radiation of a system of charges can be calculated if the motion of the charges is known because that is sufficient to determine the fields that define the Poynting vector. There is a direct relationship between the motion of a system of charges and the resulting radiation. *H.A. Lorentz* went further and extended the result for an isolated charge that is accelerated by any force (a magnetic field, gravity,etc.), *independently of the existence of electromagnetic potential energy*. This author demonstrated that the field in the proximity of a charge with spherical symmetry becomes distorted by the combined effects of the acceleration of the charge and the finite propagation velocity of changes in the field. This distortion generates a net "self-force"[3] of the field on the particle on its own source; the displacement of this force can represent, at least in certain cases, the radiated electromagnetic energy. In this way, Lorentz did not attribute radiation to the relative accelerations between the charges in the system, as would be expected if there was a relationship with the potential energy, but to the *acceleration of a charge with respect to any inertial coordinate system*. An accelerated point charge emits energy and impulses in the form of radiation. The reason for this attribution is the presence of emitted energy in the form of oscillations of the particle's electromagnetic field in regions relatively far from the particle; these make up the *radiation field*. An observer in the radiation field can relate this energy to an event that occurred at a point occupied by the charge at an earlier time. This delay time corresponds to the propagation velocity of an electromagnetic signal from the particle, and the event referred to is a change in the velocity of the charged particle. In regard to the conservation of energy, the radiated energy is directly extracted from the mechanical kinetic energy of the charged particle, not from the potential energy of the electromagnetic system.

## 2. LARMOR'S FORMULA

Consider a spherical surface in a radiation field with an accelerated charge at the centre. The amount of energy *W* that exits the surface in the form of electromagnetic radiation per unit of time is given by *Larmor's formula,*

$$\frac{dW}{dt} = \frac{q^2}{6\pi\varepsilon_0 c^3}\left[\bar{a}(t-\tau_d)\right]^2 \qquad (2.1)$$

where $q$ is the amount of electric charge, $\boldsymbol{a}$ is its acceleration and $\tau_d$ corresponds to the previously mentioned delay. The cause of the radiated energy at time $t$ is the charge acceleration at time $t$ - $\tau_d$. Following these ideas, we can search for a force whose displacement generates this radiated energy. If such a force exists and obeys Newton's laws, it must satisfy

$$\sum \bar{F}_i = m\bar{a} \Rightarrow \bar{F}_{ext} + \bar{F}_r = m\bar{a} \qquad (2.2)$$

where the subscript "*ext*" indicates the resultant of the external forces, and the subscript "*r*" indicates the Lorentz "self-force" between the particle and its own field. Assuming that the effect of this force is to decrease the kinetic energy of the particle and emit it as radiation, we can write

$$\int \bar{F}_r \bullet d\bar{r} = -\int \frac{q^2}{6\pi\varepsilon_0 c^3}\left[\bar{a}(t)\right]^2 dt \qquad (2.3)\,.$$

Note that we are now disregarding the delay. Newtonian forces are essentially local magnitudes and are from the perspective of an observer local to the particle, not from an observer located in the radiation field. In principle, this should not be a problem because we have seen that the energy that arrives in the radiation field comes from the charge's acceleration, and that energy should be, at the corresponding time, as close to the charge as we want it to be.

$$\bar{F}_r \bullet d\bar{r} = -\frac{q^2}{6\pi\varepsilon_0 c^3}\bar{a} \bullet d\bar{v} = -\frac{q^2}{6\pi\varepsilon_0 c^3}\left(d(\bar{v} \bullet \bar{a}) - \bar{v} \bullet d\bar{a}\right) \Rightarrow \left(\bar{F}_r - \frac{q^2}{6\pi\varepsilon_0 c^3}\frac{d\bar{a}}{dt}\right) \bullet d\bar{r} = \frac{q^2}{6\pi\varepsilon_0 c^3} d(\bar{v} \bullet \bar{a}) \qquad (2.4)$$

It turns out that the self-deceleration force derived by Abraham and Lorentz is the second term of the previous parenthetical expression,

$$\bar{F}_r = \frac{q^2}{6\pi\varepsilon_0 c^3}\frac{d\bar{a}}{dt} \qquad (2.5)$$

According to our development, this force can be maintained when the velocity and the acceleration are always perpendicular or when there is oscillatory motion; then, the integral average of the second term in each period is zero,

$$\int_0^T \left(\bar{F}_r - \frac{q^2}{6\pi\varepsilon_0 c^3}\frac{d\bar{a}}{dt}\right) \bullet d\bar{r} = \frac{q^2}{6\pi\varepsilon_0 c^3}\bar{v} \bullet \bar{a}\Big|_0^T = 0 \qquad (2.6)$$

If we accept all of this, then, Newton's law becomes the following in the case of an accelerated charge[1]:

$$\bar{F}_{ext} + \frac{q^2}{6\pi\varepsilon_0 c^3}\frac{d\bar{a}}{dt} = m\bar{a} \qquad (2.7)$$

When the external force is oscillatory (the unforced case) we can write, in one dimension,

$$-m\omega^2 x + \frac{q^2}{6\pi\varepsilon_0 c^3}\frac{d^3x}{dt^3} = m\frac{d^2x}{dt^2} \qquad (2.8)\,,$$

which is an equation that has been widely studied in the literature and has physically feasible consequences. Mathematical appendix includes an explanation of this eq. according the theory we will develop. However, in the case of a constant external force, the differential equation is simple, and its solution is

$$\bar{a} = \frac{\bar{F}_{ext}}{m} + \bar{a}_0 e^{t/\tau_0}; \qquad \tau_0 = \frac{q^2}{6\pi\varepsilon_0 mc^3} \qquad (2.9),$$

which predicts a constant acceleration, a self-acceleration :a spontaneous exponential increase in the acceleration. The discrepancy between this result and a real situation is radical. In charged particle accelerators, where this result can be observed, nothing similar occurs. Have we obtained a reduction to absurdity? Let us review the important points and try not to repeat the same mistakes.

## 3. THE RADIATION COMES FROM THE KINETIC ENERGY OF THE CHARGE.

Imagine an object in motion in our coordinate system. We introduce into this object a certain amount of electric charge that is small enough that the modification of its mass is negligible; we place the object in an external magnetic field (that is exclusively magnetic). Due to the *Lorentz magnetic force*, the object experiences a force normal to its trajectory, and due to the acceleration generated, the charge starts to emit electromagnetic radiation. The radiation is emitted at the expense of the object's kinetic energy and ceases when the object is at rest in our coordinate system. This phenomenon has been confirmed in certain particle accelerators (cyclotrons),and therefore, there is experimental support for our assumption that kinetic energy can be completely transformed into radiation.

In the previous section, we used Larmor's formula but used instantaneous rather than delayed acceleration to introduce a force associated with the radiation. In the following argument, we reason about the hypothesis that the radiation comes from the kinetic energy *without resorting to the possibility that this force exists*. Suppose a force is applied to a charge that is at rest. From the very definition of acceleration ($a$), we know that the change in the velocity ($v$) of the particle at time $dt$ is ***dv=a*dt***; however, if at that time, the charge emits a certain amount of energy while *keeping its mass m constant* at the expense of its own kinetic energy $E_c$, we can no longer be confident of the definition of acceleration. In fact, from the definition of acceleration, it follows that $d\bar{v} = \bar{a}\,dt \Rightarrow m\bar{v}\bullet d\bar{v} = m\bar{v}\bullet \bar{a}\,dt \Rightarrow dE_c = \bar{v}\bullet m\bar{a}\,dt$ ; that is, for a given instantaneous acceleration, the instantaneous variation in the kinetic energy, $dE_c$, is completely determined if we suppose that the mass ,*m,* of the particle is constant whether the particle is charged or not. If the radiation implies consumption of kinetic energy , then $dE_c \neq \bar{v}\bullet m\bar{a}\,dt$, which conflicts with the definition of instantaneous acceleration. Evidently, this situation has led us into a very basic analytical error because we can not disregard the definition of instantaneous acceleration, and therefore, we must conclude that:

1. *An analysis that uses Larmor's formula must include the effect of the delayed acceleration.*

2. *An analysis of the motion of an accelerated charge must consider the possibility of fluctuations in the mechanical mass.*

A non-inertial accelerated observer that perceives a charge at permanent rest and without mass fluctuations does not observe a change in the kinetic energy, and therefore, according to our hypothesis, does not observe the emission of radiation. If the laws of physics must be valid for any observer, then the mass fluctuation may be a valid explanation of the radiation perceived by any observer. Note that with this approach, the existence of a radiation force is still possible, but such a force is a consequence of the mass fluctuations of the particle.

## 4. POYNTING'S THEOREM FOR AN ACCELERATED CHARGE.

Following the observations above, consider a spherical surface centred on the charge at time *dt*. The sphere's radius ($c\tau_d$), being $c$ light's velocity in vacuum and $\tau_d$ a time delay, is large enough to suppose that it reaches the radiation field but small enough to use a linear approximation for the delayed acceleration resulting from the charge's instantaneous acceleration. Applying Poynting's theorem to this limited volume $V$ yields

$$-\frac{d}{dt}\left(\frac{\varepsilon}{2}\int E^2 dV + \frac{1}{2\mu}\int B^2 dV\right) = \int \rho\bar{v}\bullet\bar{E}dV + \int\left(\bar{E}\times\bar{H}\right)\bullet d\bar{S} \qquad (4.1)$$

If we consider volume divided into integration elements,$dV$, and bordered by a surface divided into elements,$dS$, we have the following concepts:

1-*The first term* is the rate of change of the amount of electromagnetic energy contained in the volume. In the case of a point charge accelerated by an external field, we can decompose the field into the sum of the external field and the particle field as follows:

$$\frac{\varepsilon}{2}\int\left(\bar{E}_{ext}+\bar{E}_{par}\right)^2 dV + \frac{1}{2\mu}\int\left(\bar{B}_{ext}+\bar{B}_{par}\right)^2 dV = \qquad (4.2)$$
$$\frac{\varepsilon}{2}\int E_{ext}{}^2 dV + \frac{1}{2\mu}\int B_{ext}{}^2 dV + \frac{\varepsilon}{2}\int E_{par}{}^2 dV + \frac{1}{2\mu}\int B_{par}{}^2 dV + \varepsilon\int\bar{E}_{ext}\bullet\bar{E}_{par}dV + \frac{1}{\mu}\int\bar{B}_{ext}\bullet\bar{B}_{par}dV$$

The summands that depend only on one field correspond to the external field's own energy and the *charge's electromagnetic mass* in the volume to be integrated over. The summands that depend on the scalar product of two different fields correspond to the *interaction energy* between the particle and the external field.

2-*The second term* includes the charge density, the velocity and the electric field. The Lorentz force on a point charge, allows this term to be written as the rate of change of the kinetic energy. For a point charge distribution with mechanic impulse $p$ and kinetic energy $E_c$

$$\int\rho\bar{v}\bullet\bar{E}\,dV = \int\bar{v}\bullet\left(\bar{E}+\bar{v}\times\bar{B}\right)\rho dV = \int\bar{v}\bullet\frac{d^2\bar{p}}{dtdV}dV = \int\frac{d}{dV}\left(\bar{v}\bullet\frac{d\bar{p}}{dt}\right)dV = \bar{v}\bullet\frac{d\bar{p}}{dt} = \frac{dE_c}{dt} \qquad (4.3)$$

3-*The third term* corresponds to the flow of electromagnetic energy per unit time that leaves the integration surface. Suppose that no particles travel through this border; otherwise a term associated with the flow of mechanical energy through the surface must be included. Under these conditions, and in the case of an accelerated point charge, this term corresponds to Larmor's formula,

$$\int\left(\bar{E}\times\bar{H}\right)\bullet d\bar{S} = \frac{q^2}{6\pi\varepsilon_0 c^3}\left[\bar{a}(t-\tau_d)\right]^2;\ c\tau_d = sphere-radius \qquad (4.4)$$

Summarizing the above into an equation with the assumption of a <u>*constant external field*</u>, we have

$$-\frac{d}{dt}\left(\varepsilon\int\bar{E}_{ext}\bullet\bar{E}_{par}dV + \frac{1}{\mu}\int\bar{B}_{ext}\bullet\bar{B}_{par}dV\right) = \frac{d}{dt}\left(\frac{\varepsilon}{2}\int E_{par}{}^2 dV + \frac{1}{2\mu}\int B_{par}{}^2 dV + E_c\right) + \frac{q^2}{6\pi\varepsilon_0 c^3}\left[\bar{a}(t-\tau_d)\right]^2 \qquad (4.5)$$

In regard to the particle's field, the theoretical results indicate that there are two component fields[1]:

***1-$E^p_{par}$, $B^p_{par}$***:A quasi-stationary field that is the same as the field of a point charge that moves at a constant velocity but that depends on the delayed velocity. The electric field lines go through the charged particle.

***2-$E^r_{par}$, $B^r_{par}$***:A radiation field that is independent of the previous one. The lines of this field do not go through the charged particle.

Consequently, we have

$$\frac{\varepsilon}{2}\int E_{par}{}^2 dV = \frac{\varepsilon}{2}\int\left(\overline{E}^p_{par}+\overline{E}^r_{par}\right)^2 dV = \frac{\varepsilon}{2}\int\left(\overline{E}^p_{par}\right)^2 dV + \frac{\varepsilon}{2}\int\left(\overline{E}^r_{par}\right)^2 dV + \varepsilon\int\left(\overline{E}^p_{par}\bullet\overline{E}^r_{par}\right)dV \qquad (4.6.1)$$

$$\frac{1}{2\mu}\int B_{par}{}^2 dV = \frac{1}{2\mu}\int\left(\overline{B}^p_{par}+\overline{B}^r_{par}\right)^2 dV = \frac{1}{2\mu}\int\left(\overline{B}^p_{par}\right)^2 dV + \frac{1}{2\mu}\int\left(\overline{B}^r_{par}\right)^2 dV + \frac{1}{\mu}\int\left(\overline{B}^p_{par}\bullet\overline{B}^r_{par}\right)dV \qquad (4.6.2)$$

In *The Feynman Lectures on Physics* [3], the reader can find an explanation of the *electromagnetic mass*, $m_e$, and see that it implies that particles are actually objects of a defined size that can vary. Under these conditions, for velocities that are much lower than the speed of light, we have

$$m_e c^2 = \frac{\varepsilon}{2}\int\left(E^p_{par}\right)^2 dV \;\; ; \; \frac{1}{2}m_e v^2 = \frac{1}{2\mu}\int\left(B^p_{par}\right)^2 dV \qquad (4.7.)$$

In principle, values of the electromagnetic mass are defined for stationary fields; however, we can assume that the fields evolve relatively slowly so that a quasi-static approximation is applicable, which enables us to express Poynting's theorem as:

$$-\frac{d}{dt}\left(\varepsilon\int\overline{E}_{ext}\bullet\overline{E}^p_{par}dV + \frac{1}{\mu}\int\overline{B}_{ext}\bullet\overline{B}^p_{par}dV\right) - \frac{d}{dt}\left(\varepsilon\int\overline{E}_{ext}\bullet\overline{E}^r_{par}dV + \frac{1}{\mu}\int\overline{B}_{ext}\bullet\overline{B}^r_{par}dV\right) = \qquad (4.8)$$

$$\frac{d}{dt}\left(m_e c^2 + \frac{1}{2}m_e v^2 + E_c\right) + \frac{d}{dt}\left(\frac{\varepsilon}{2}\int\left(\overline{E}^r_{par}\right)^2 dV + \frac{\varepsilon}{2}\int\left(\overline{B}^r_{par}\right)^2 dV\right) + \frac{d}{dt}\left(\varepsilon\int\left(\overline{E}^p_{par}\bullet\overline{E}^r_{par}\right)dV + \frac{1}{\mu}\int\left(\overline{B}^p_{par}\bullet\overline{B}^r_{par}\right)dV\right) + \frac{q^2}{6\pi\varepsilon_0 c^3}\left[\overline{a}(t-\tau_d)\right]^2$$

In this result, the left hand side of the equation includes a sum of terms associated with the interaction of the external field with the particle's own field and with the interaction between the external field and the particle's radiation field. On the right hand side of the equation, we have terms that are associated with the kinetic energy, including the electromagnetic mass, and terms associated with the radiation field. Among these, a term that represents the interaction between the charge's own field and the radiation field appears; in principle, this is the term that is related to the self-deceleration force.

## 5. THE DYNAMICS OF AN ACCELERATED POINT CHARGE. CAUSALITY.

After the introduction of the electromagnetic mass, it seems necessary to attribute an internal structure to that which we have considered a point particle. However, we introduced the kinetic energy of a point particle, and logical coherence tells us that it should be possible to continue the line of reasoning using this hypothesis. In this sense, we introduce the following relationships:

1-The variation of the energy of the radiation field inside the integration sphere: It seems evident that this variation is calculated by taking the difference between the radiated energy at time $dt$ close to the charge, which we can associate with the instantaneous acceleration, and the energy that is lost when it travels through the surface of radius $c\tau_d$, which is associated with the delayed acceleration, as follows:

$$\frac{d}{dt}\left(\frac{\varepsilon}{2}\int\left(\overline{E}^r_{par}\right)^2 dV + \frac{\varepsilon}{2}\int\left(\overline{B}^r_{par}\right)^2 dV\right) = \frac{q^2}{6\pi\varepsilon_0 c^3}\left[\overline{a}(t)\right]^2 - \frac{q^2}{6\pi\varepsilon_0 c^3}\left[\overline{a}(t-\tau_d)\right]^2 \qquad (5.1)$$

2-The variation in the energy in the particle's own field, which is related to the electromagnetic mass, inside the integration sphere: In the *Berkeley Physics Course* on *Electromagnetism* [5], the reader can find an intuitive calculation of the energy variation of a particle's own field in the appendix on the radiation of an accelerated charge; that is, the field lines go through the charged particle. According to this reference, we can write the following result, which is equivalent to the above equation:

$$\frac{d}{dt}\left(m_e c^2 + \frac{1}{2} m_e v^2\right) = \frac{q^2}{6\pi\varepsilon_0 c^3}\left[\bar{a}(t)\right]^2 - \frac{q^2}{6\pi\varepsilon_0 c^3}\left[\bar{a}(t-\tau_d)\right]^2 \qquad (5.2)\,.$$

Substituting this into the equation we derived using Poynting's theorem, we obtain

$$\begin{aligned} &-\frac{d}{dt}\left(\varepsilon\int \bar{E}_{ext} \bullet \bar{E}^{p}_{par} dV + \frac{1}{\mu}\int \bar{B}_{ext} \bullet \bar{B}^{p}_{par} dV\right) - \frac{d}{dt}\left(\varepsilon\int \bar{E}_{ext} \bullet \bar{E}^{r}_{par} dV + \frac{1}{\mu}\int \bar{B}_{ext} \bullet \bar{B}^{r}_{par} dV\right) = \qquad (5.3)\,. \\ &\frac{dE_c}{dt} + \frac{q^2}{6\pi\varepsilon_0 c^3}\left[\left[\bar{a}(t)\right]^2 + \left(\left[\bar{a}(t)\right]^2 - \left[\bar{a}(t-\tau)\right]^2\right)\right] + \frac{d}{dt}\left(\varepsilon\int\left(\bar{E}^{p}_{par} \bullet \bar{E}^{r}_{par}\right) dV + \frac{1}{\mu}\int\left(\bar{B}^{p}_{par} \bullet \bar{B}^{r}_{par}\right) dV\right) \end{aligned}$$

A first approximation of the acceleration terms is

$$\left[\bar{a}(t)\right]^2 + \left(\left[\bar{a}(t)\right]^2 - \left[\bar{a}(t-\tau_d)\right]^2\right) \approx \left[\bar{a}(t)\right]^2 + 2\bar{a} \bullet \frac{d\bar{a}}{dt}\tau_d \qquad (5.4)\,,$$

but note that this first approximation is the same as the approximation of the square of the acceleration not delayed but advanced.

$$\left[\bar{a}(t+\tau_d)\right]^2 \approx \left[\bar{a}(t)\right]^2 + 2\bar{a} \bullet \frac{d\bar{a}}{dt}\tau_d \qquad (5.5)\,.$$

This situation can affect the principle of causality because even to a first approximation, we expect results that are consistent with causality. When introducing, as a first approximation in the principle of energy conservation, the value of a parameter at a future time, the evolution of the physical system depends on the future conditions under which, in accordance with the principle of causality, we suppose that cause always precede effects. To avoid this situation, Poynting's equation must provide us with an alternative that removes the "non-causal" term in the first approximation. We can see that the integral associated with the interaction between the radiation field and the particle's own field can do this. If we assume that the interaction between the external field and the radiation field is also negligible, or we include it in the corresponding term (the first eq.), decoupling Poynting's equation generates two other equations. According to our approach, we consider the parameter $\tau_d$ approximately constant so that

$$-\frac{d}{dt}\left(\varepsilon\int \bar{E}_{ext} \bullet \bar{E}^{p}_{par} dV + \frac{1}{\mu}\int \bar{B}_{ext} \bullet \bar{B}^{p}_{par} dV\right) = \frac{dE_c}{dt} + \frac{q^2}{6\pi\varepsilon_0 c^3}\left[\bar{a}(t)\right]^2 \qquad (5.6)$$

$$\frac{d}{dt}\left(\varepsilon\int\left(\bar{E}^{p}_{par} \bullet \bar{E}^{r}_{par}\right) dV + \frac{1}{\mu}\int\left(\bar{B}^{p}_{par} \bullet \bar{B}^{r}_{par}\right) dV + \frac{q^2}{6\pi\varepsilon_0 c^3}\left[\bar{a}(t)\right]^2 \tau_d\right) = 0 \qquad (5.7)$$

.

It is straightforward to interpret eq. (5.6) according to the principle of conservation of energy: at time *dt*, the first term corresponds to the energy transferred by the external force, and the second is the sum of the kinetic energy variations and the radiated energy. Equation (5.7) corresponds to a coupling between the charge and the radiation emitted by it so that the energy associated with the coupling between the radiation field and the particle's own field is directly related to the radiation process of the particle. To calculate this integral directly, it is necessary to know the internal structure of the particle; however, the development that we have followed raises the need for this coupling integral to have a determined and non-negligible value in the first approximation. Formally, the previous result corresponds to a *constant of motion* with dimensions of energy or mass. In the line of this study, we interpret this relationship as a way of compensating so that the mass of the particle remains constant. We can see that *the causality condition lead us to the mass constancy.*

If by following classical mechanics, as in the second eq.(5.7), we introduce a force $\boldsymbol{f}$ for this coupling, we can write

$$\bar{f} \bullet \bar{v} + \frac{q^2}{3\pi\varepsilon_0 c^3}\frac{d\bar{a}}{dt} \bullet \bar{a}\tau_d = 0 \qquad (5.8)\,.$$

When the particle is at rest at time $t$-$\tau_d$, we have $\boldsymbol{a}\tau_d \approx \boldsymbol{v}$, and we can estimate the coupling force as

$$\bar{f} = -\frac{q^2}{3\pi\varepsilon_0 c^3}\frac{d\bar{a}}{dt} \qquad (5.9)\,.$$

Aside from a factor of ½ and the minus sign, this force is similar to the one given in equation (2.5).

For an inertial observer at instantaneous rest with respect to the accelerated charge, the radiation emitted by the charge at that time is emitted with symmetry such that its mechanical impulse is null [2]. When the relativistic transformation of the mechanical impulse is applied, we obtain

$$\Delta P_l = 0 = \frac{\Delta p - \frac{v}{c^2}\Delta\varepsilon}{\sqrt{1-\frac{v^2}{c^2}}} \qquad (5.11)\,,$$

and therefore, for any other inertial observer, $\Delta\boldsymbol{p}=\boldsymbol{v}\Delta\varepsilon/c^2$, where $\boldsymbol{p}$ and $\varepsilon$ are the mechanical impulse and the energy of the emitted radiation, respectively, and $\boldsymbol{v}$ is the particle's velocity. If we assume that the radiation comes from the particle's kinetic energy, we can then take [$\Delta\boldsymbol{p}$, $\Delta\varepsilon$] as components of the particle's energy-impulse. If we divide (5.11) by the corresponding time, $\Delta t$, and we include all of these in the equation (5.9) for the radiation force and use the Larmor's fórmula, we obtain in the limit

$$\bar{f} = \frac{d\bar{p}}{dt} = \frac{\bar{v}}{c^2}\frac{d\varepsilon}{dt} = -\frac{q^2}{3\pi\varepsilon_0 c^3}\frac{d\bar{a}}{dt} \qquad \Rightarrow \qquad \frac{\bar{v}}{c^2}a^2 + \frac{d\bar{a}}{dt} = 0 \quad (5.12)\,.$$

Note that according to the fundamental dynamic equation, a force that is proportional to the velocity is a sign of a variation in the mass,

$$\frac{d\bar{p}}{dt} = \frac{d}{dt}(m\bar{v}) = \bar{v}\frac{dm}{dt} + m\frac{d\bar{v}}{dt} \quad (5.13)$$

From this perspective, we see that the effect of the radiation coupling force is to restore the lost mass. Taking the scalar product of eq. (5.12) and the acceleration, $dv/dt$, we obtain

$$\frac{d\bar{v}}{dt}\bullet\frac{d^2\bar{v}}{dt^2} + \left(\frac{a}{c}\right)^2\bar{v}\bullet\frac{d\bar{v}}{dt} = 0 \Rightarrow \frac{d}{dt}\left[\left(\frac{d\bar{v}}{dt}\right)^2 + \int_0^t\left(\frac{a}{c}\right)^2 d\left(v^2\right)\right] = 0 \qquad (5.14)\,.$$

Note the similarity of this equation with eq. (5.7). This similarity suggests that eq. (5.13) is only applicable when the external fields are constant. It is evident that if the radiation phenomenon did not occur, the acceleration of the particle in a constant electric field would be constant, $\boldsymbol{a}=q\boldsymbol{E}/m$, *neglecting relativistic effects*. Therefore, in the case of a constant electric field, the only cause of *variations in the acceleration* is the emission of radiation. If the field is not constant, it is evident that the particle's acceleration can vary in response to the external field and that the radiation is only responsible for variations in the difference between the real acceleration of the particle and the acceleration caused by the external field. Therefore, we generalize eq. (5.13) as follows:

$$\frac{d}{dt}\left(\bar{a} - \frac{q}{m}\left[\bar{E}_{ext} + \bar{v}\times\bar{B}_{ext}\right]\right) + \left(\frac{a}{c}\right)^2\bar{v} = 0 \qquad (5.15)$$

From eq. (5.15)[1]for constant fields, we can determine the following integrals immediately:

[1]The final form of this equation will be shown in the section entitled *Quantum interpretation*.

$$\bar{a}\bullet\left(\frac{d\bar{a}}{dt}-\frac{q}{m}\left[\bar{a}\times\bar{B}_{ext}\right]\right)+\left(\frac{a}{c}\right)^{2}\bar{a}\bullet\bar{v}=0\Rightarrow\frac{da^{2}}{dt}+\left(\frac{a}{c}\right)^{2}\frac{dv^{2}}{dt}\Rightarrow\frac{da^{2}/dt}{a^{2}}+\frac{1}{c^{2}}\frac{dv^{2}}{dt}=0\Rightarrow\ln\left(\frac{a}{a_{0}}\right)=-\frac{1}{2c^{2}}\left(v^{2}-v_{0}^{2}\right)\Rightarrow a=a_{0}\exp\left(-\frac{v^{2}-v_{0}^{2}}{2c^{2}}\right)\quad(5.16)$$

$$\bar{v}\times\frac{d\bar{a}}{dt}-\frac{q}{m}\bar{v}\times\left[\bar{a}\times\bar{B}_{ext}\right]+\left(\frac{a}{c}\right)^{2}\bar{v}\times\bar{v}=0\Rightarrow\bar{v}\times\frac{d\bar{a}}{dt}-\frac{q}{m}\bar{v}\times\left[\bar{a}\times\bar{B}_{ext}\right]=0\Rightarrow\frac{d}{dt}\left(\bar{v}\times\bar{a}+\frac{q}{m}\frac{v^{2}}{2}\bar{B}_{ext}\right)=\frac{q}{m}\bar{a}\left(\bar{B}_{ext}\bullet\bar{v}\right)\quad(5.17)$$

According to eq. (5.17), when $\boldsymbol{B}=0$ or $\boldsymbol{B}$ and $\boldsymbol{v}$ are perpendicular, there is a constant vector that is perpendicular to the instantaneous velocity, and therefore, the motion develops on a plane. If $\boldsymbol{B}$ and $\boldsymbol{v}$ are not perpendicular, then, the motion cannot be considered planar. Continuing the derivation using the result given in eq. (5.16), we obtain

$$\frac{da^{2}}{dt}+\left(\frac{a}{c}\right)^{2}\frac{dv^{2}}{dt}=0\Rightarrow\qquad\frac{d^{2}a^{2}}{dt^{2}}+\left(\frac{a}{c}\right)^{2}\frac{d^{2}v^{2}}{dt^{2}}+\frac{1}{c^{2}}\frac{dv^{2}}{dt}\frac{da^{2}}{dt}=0\qquad(5.18)$$

which tells us that, if they exist, the extreme values of the square of the acceleration and the square of the velocity are related, i.e., a maximum of $\boldsymbol{a}^2$ corresponds to a minimum of $\boldsymbol{v}^2$ and vice versa. However, note that eq. (5.18) does not establish the existence of these extremes.Note that according to our line of argument, in principle, eq. (5.6) for the conservation of energy is only valid when the external fields are constant. This suggests approximating the power transferred to the charge as

$$q\bar{E}_{ext}\bullet\bar{v}=\frac{dE_{c}}{dt}+\frac{q^{2}}{6\pi\varepsilon_{0}c^{3}}a^{2}=m\bar{a}\bullet\bar{v}+\frac{q^{2}}{6\pi\varepsilon_{0}c^{3}}a^{2}\quad(5.19)$$

Recalling that the kinetic energy is proportional to $\boldsymbol{v}^2$, we see that the kinetic energy minimum in eq. (5.19) must result in a non-zero velocity so that the previous expression does not result in a kinetic energy minimum of zero. We can imagine a positive charge that initially moves against an electric field. It is evident that the deceleration due to the field ensures that the kinetic energy reaches a minimum at some time, and if the motion is rectilinear, it is not easy to understand why the kinetic energy cannot become zero at some time. In this way, we have not been able to give a concrete solution to eq. (5.6) by following a classical description of the dynamics of an accelerated charge. However, eq. (5.15) does allow a dynamic solution in a simple case, and the results can be contrasted with eq. (5.19), as we will see in the following section.

## 6. Rectilinear Motion Of A Point Charge In A Constant Electric Field. Brownian Motion?

In this case and for a positive charge, the velocity, the acceleration and the electric field maintain the same constant direction so that eq. (5.15) can be written in terms of modules, and because the acceleration does not cancel out, we can divide the expression and integrate it as follows:

$$\frac{1}{a^{2}}\frac{da}{dt}+\frac{v}{c^{2}}=0\Rightarrow-\frac{1}{a}+\frac{x}{c^{2}}=-\frac{m}{qE}\qquad(6.1)$$

where $x$ is the position of the particle in the direction of motion ;initially, $x=0$ and the acceleration is Newtonian. From the previous equation we obtain

$$ma=\frac{qE}{1+\frac{qEx}{mc^{2}}}\Rightarrow\int ma\,dx=\int\frac{qE}{1+\frac{qEx}{mc^{2}}}dx\Rightarrow\frac{v^{2}}{2}-\frac{v_{0}^{2}}{2}=c^{2}\ln\left(1+\frac{qEx}{mc^{2}}\right)\quad(6.2)$$

Approximating the logarithm of a *small displacement with respect to x=0 or for* ***E*** *close to zero*, we obtain the expression

$$qEx\approx\frac{1}{2}mv^{2}-\frac{1}{2}mv_{0}^{2}+\frac{1}{2}\left(\frac{q^{2}E^{2}}{mc^{2}}\right)x^{2}-\frac{1}{3}\left(\frac{q^{3}E^{3}}{m^{2}c^{4}}\right)x^{3}+....\quad(6.3),$$

where we immediately recognise the form of eq. (5.19) with the potential and kinetic energies, and by cancelling terms, the energy emitted in the radiation is

$$\frac{1}{2}\left(\frac{q^2E^2}{mc^2}\right)x^2-\frac{1}{3}\left(\frac{q^3E^3}{m^2c^4}\right)x^3+\ldots\approx\frac{q^2}{6\pi\varepsilon_0c^3}a^2t \quad (6.4),$$

and the acceleration, $a$, has an average value. At the limit in which the electric field tends to zero, we assume that the average acceleration also tends to the Newtonian value: as $\boldsymbol{E}\rightarrow 0, \boldsymbol{a}\rightarrow q\boldsymbol{E}/m$. Therefore, we consider the following limit

$$Lim_{E\to 0}\frac{\frac{1}{2}\left(\frac{q^2E^2}{mc^2}\right)x^2-\frac{1}{3}\left(\frac{q^3E^3}{m^2c^4}\right)x^3+\ldots}{\frac{q^2a^2}{6\pi\varepsilon_0c^3}t}=\frac{3\pi\varepsilon_0mcx^2}{tq^2}=1\Rightarrow x^2\approx\frac{q^2}{3\pi\varepsilon_0mc}t \quad (6.5)$$

Note the reader that the above limit, if any, should be independent of how it reaches him : taking the field $\boldsymbol{E}$ in the $+x$ direction or the opposite $-x$. This consideration suggests that the most appropriate is that the value found $x^2$ corresponds to a mean square value of a particle that can move with equal probability in both directions $+x$ and -x. Thus equation (6.5) is similar to the *brownian motion* of a particle with a constant average kinetic energy. Therefore, even though it performs an intrinsic behaviour, a free charged particle appears as if it were moving in a viscous medium and affected by random impacts.

## 7. QUANTUM INTERPRETATION.

Equation (5.15) is not physically satisfactory because the mechanical impulse of the radiation should be proportional to the charge, $q$, as is the case of the radiated energy in Larmor's formula. We generalize the equation by introducing a dimensionless factor $\beta$ as follows:

$$\boxed{\frac{d}{dt}\left(\bar{a}-\frac{q}{m}\left[\bar{E}_{ext}+\bar{v}\times\bar{B}_{ext}\right]\right)+\beta\left(\frac{a}{c}\right)^2\bar{v}=0 \quad (7.1)},$$

which is the dynamic equation for an accelerated charge. The mathematical consequences of this formula are the same as those of eq. (5.15) with the change $c^2\rightarrow\beta^{-1}c^2$. In the limiting case in which a constant electric field tends toward zero in eq. (6.5), the result becomes

$$a=a_0\exp\left(-\beta\frac{v^2-v_0^2}{2c^2}\right) \quad (7.2)$$

In the limiting case in which a constant electric field tends toward zero in eq. (6.5), the result becomes

$$Lim_{E\to 0}\frac{\frac{1}{2}\left(\frac{q^2E^2}{m\beta^{-1}c^2}\right)x^2-\frac{1}{3}\left(\frac{q^3E^3}{m^2\beta^{-2}c^4}\right)x^3+\ldots}{\frac{q^2a^2}{6\pi\varepsilon_0c^3}t}=\frac{3\pi\varepsilon_0mcx^2}{tq^2\beta^{-1}}=1\Rightarrow x^2\approx\frac{1}{\beta}\frac{q^2}{3\pi\varepsilon_0mc}t \quad (7.3).$$

When we derive the result, we obtain

$$mv\,x\approx\frac{1}{\beta}\frac{q^2}{6\pi\varepsilon_0c} \quad (7.4)$$

Interpreting this equation according to *Heisenberg's Uncertainty Principle* reveals that the following constant is an admissible value for $\beta$:

$$\beta = \frac{q^2}{\hbar c 4\pi\varepsilon_0};\ \Rightarrow mv\,x = \Delta p \Delta x \approx \hbar \qquad (7.5),$$

where $\hbar$ is Planck's constant. In addition, the reader can observe that for the case of the elementary charge*(e)* $\beta$ is the fine-structure constant, $\alpha$, which yields

$$\beta = \frac{q^2}{\hbar c 4\pi\varepsilon_0} = Z^2 \frac{e^2}{\hbar c 4\pi\varepsilon_0} = Z^2\alpha \quad (7.6),$$

where $Z$ is the number of elementary charges, $e$, contained in the charge, $q$. Consequently, in eq. (5.15), the square of the charge is a factor of the radiation impulse term. Therefore, if we have two charges with the same velocity and kinematic acceleration, the one with more charge emits more radiation, which is in agreement with Larmor's formula.

## 8. Analytical Compatibility Of The Equations.

Generalizing eq.s (5.19) and (7.1) for a force ***f****(t)*, we obtain the following system of equations:

$$\left.\begin{aligned} &\left(m\bar{a} - \bar{f}\right)\bullet\bar{v} + m\tau_0 a^2 = 0 && (8.1)\\ &\frac{d}{dt}\left(\bar{a} - \frac{\bar{f}}{m}\right) + \beta\left(\frac{a}{c}\right)^2\bar{v} = 0 && (8.2)\end{aligned}\right\} \ ; \quad \tau_0 = \frac{q^2}{6\pi\varepsilon_0 c^3 m}.$$

If we derive the first energy equation with respect to time and replace the first term of the second equation in the result, we obtain

$$\bar{v}\bullet\frac{d}{dt}\left(m\bar{a} - \bar{f}\right) + \bar{a}\bullet\left(m\bar{a} - \bar{f}\right) + 2m\tau_0\bar{a}\bullet\frac{d\bar{a}}{dt} = 0 \Rightarrow \bar{a}\bullet\left[m\bar{a}\left(1 - \beta\frac{v^2}{c^2}\right) - \bar{f} + 2m\tau_0\frac{d\bar{a}}{dt}\right] = 0 \Rightarrow.$$

$$\bar{f} = m\bar{a}\left(1 - \beta\frac{v^2}{c^2}\right) + 2m\tau_0\frac{d\bar{a}}{dt} \qquad (8.3)$$

It is clear that this last equation is not compatible with the result given in eq. (5.16) when *f* is a constant force. If the accelerated particle starts at rest, its velocity increases over time, and according to eq. (5.16), its acceleration decreases over time and the derivative of the acceleration is both negative and decreasing over time. In contrast, in the previous expression, if the velocity increases over time and the acceleration decreases over time, the derivative of the acceleration can only increase over time. Therefore, eq.s (8.1) and (8.2) are incompatible. A possible solution is that this incompatibility is due to relativistic effects that have not been considered in the energy equation although the dynamic equation already includes them. Let us derive a first relativistic correction to the kinetic energy

$$\bar{p} \approx m\bar{v}\left(1 + \frac{v^2}{2c^2}\right) \Rightarrow \frac{d\bar{p}}{dt} \approx m\bar{a}\left(1 + \frac{3}{2}\frac{v^2}{c^2}\right) - \frac{m}{c^2}\left(\bar{v}\times\left(\bar{a}\times\bar{v}\right)\right) \Rightarrow \frac{dE_c}{dt} = \bar{v}\bullet\frac{d\bar{p}}{dt} \approx m\bar{v}\bullet\bar{a}\left(1 + \frac{3}{2}\frac{v^2}{c^2}\right) \qquad (8.4),$$

and for the power radiated, we have

$$\frac{dW}{dt} \approx m\tau_0\left(1 + 3\frac{v^2}{c^2}\right)\left(a^2 - \frac{1}{c^2}\left(\bar{v}\times\bar{a}\right)^2\right) \qquad (8.5).$$

Therefore, the system of equations is now

$$\left.\begin{aligned} &\left[\left(m\bar{a} - \bar{f}\right) + \frac{3}{2}m\frac{v^2}{c^2}\bar{a}\right]\bullet\bar{v} + m\tau_0\left(1 + 3\frac{v^2}{c^2}\right)\left(a^2 - \frac{1}{c^2}\left(\bar{v}\times\bar{a}\right)^2\right) = 0 && (8.6)\\ &\frac{d}{dt}\left(\bar{a} - \frac{\bar{f}}{m}\right) + \beta\left(\frac{a}{c}\right)\bar{v} = 0 && (8.7)\end{aligned}\right\}$$

By deriving the first equation and using the second equation, we obtain

$$\left[-m\frac{\beta}{c^2}a^2\bar{v}+\frac{3m}{2}\frac{v^2}{c^2}\frac{d\bar{a}}{dt}+\frac{3m}{c^2}\left(\bar{v}\bullet\bar{a}\right)\bar{a}\right]\bullet\bar{v}+\left[m\bar{a}\left(1+\frac{3}{2}\frac{v^2}{c^2}\right)-\bar{f}\right]\bullet\bar{a}+2m\tau_0\left(1+3\frac{v^2}{c^2}\right)\bar{a}\bullet\frac{d\bar{a}}{dt}+m\tau_0\frac{6\bar{v}\bullet\bar{a}}{c^2}a^2=0\Rightarrow \qquad (8.8)$$

$$\bar{v}\bullet\left[\frac{3m}{2}\frac{v^2}{c^2}\frac{d\bar{a}}{dt}\right]+\bar{a}\bullet\left[\frac{3m}{c^2}\left(\bar{v}\bullet\bar{a}\right)\bar{v}+m\bar{a}\left(1+\left[\frac{3}{2}-\beta\right]\frac{v^2}{c^2}\right)-\bar{f}+2m\tau_0\left(1+3\frac{v^2}{c^2}\right)\left(\frac{d\bar{a}}{dt}+\frac{\bar{v}\times\left(\bar{v}\times\frac{d\bar{a}}{dt}\right)}{c^2}\right)+6m\tau_0\frac{\left(a^2-\frac{1}{c^2}\left(\bar{v}\times\bar{a}\right)^2\right)}{c^2}\bar{v}\right]=0$$

Disregarding terms that contain $\boldsymbol{v^2/c^2}$ (or that are lower by a factor of *sine* or *cosine*), as a first approximation, we obtain

$$\bar{f}\approx m\bar{a}+6m\tau_0\frac{a^2}{c^2}\bar{v}+2m\tau_0\frac{d\bar{a}}{dt} \qquad (8.9)$$

Note that the first two summands of the second term can be interpreted as the derivative of the mechanical impulse ($\boldsymbol{p=mv}$) with respect to time, which includes an increase in the mechanical mass. Removing the second summand of the right hand side, based on eq. (8.7), we obtain

$$\bar{f}\approx m\bar{a}+2m\tau_0\frac{d\bar{a}}{dt}-\frac{6m\tau_0}{\beta}\frac{d}{dt}\left(\bar{a}-\frac{\bar{f}}{m}\right)\Rightarrow\bar{f}-\frac{6\tau_0}{\beta}\frac{d\bar{f}}{dt}\approx m\bar{a}+2m\tau_0\left(1-\frac{3}{\beta}\right)\frac{d\bar{a}}{dt} \qquad (8.10).$$

We can integrate this result by multiplying by an exponential factor, *exp{(t-τ)/τ₁}*, taking $\tau_1$ conveniently and integrating by parts as follows

$$\tau_1=\frac{6\tau_0}{\beta}\Rightarrow\bar{f}(\tau)\exp\left(\frac{t-\tau}{\tau_1}\right)-\tau_1\frac{d\bar{f}}{d\tau}\exp\left(\frac{t-\tau}{\tau_1}\right)=m\bar{a}\exp\left(\frac{t-\tau}{\tau_1}\right)-m\tau_1\frac{d\bar{a}}{d\tau}\exp\left(\frac{t-\tau}{\tau_1}\right)+2m\tau_0\frac{d\bar{a}}{dt}\exp\left(\frac{t-\tau}{\tau_1}\right)\Rightarrow$$

$$\bar{f}(\tau)\exp\left(\frac{t-\tau}{\tau_1}\right)-\tau_1\left\{\frac{d}{d\tau}\left(\bar{f}(\tau)\exp\left(\frac{t-\tau}{\tau_1}\right)\right)+\frac{1}{\tau_1}\bar{f}(\tau)\exp\left(\frac{t-\tau}{\tau_1}\right)\right\}=m\bar{a}\exp\left(\frac{t-\tau}{\tau_1}\right)-m\tau_1\left\{\frac{d}{d\tau}\left(\bar{a}(\tau)\exp\left(\frac{t-\tau}{\tau_1}\right)\right)+\frac{1}{\tau_1}\bar{a}(\tau)\exp\left(\frac{t-\tau}{\tau_1}\right)\right\}+2m\tau_0\frac{d\bar{a}}{d\tau}\exp\left(\frac{t-\tau}{\tau_1}\right)\Rightarrow$$

$$\frac{d}{d\tau}\left(\left\{m\bar{a}-\bar{f}(\tau)\right\}\exp\left(\frac{t-\tau}{\tau_1}\right)\right)=\frac{2m\tau_0}{\tau_1}\frac{d\bar{a}}{d\tau}\exp\left(\frac{t-\tau}{\tau_1}\right)$$

$$\int_t^{\infty}\frac{d}{d\tau}\left(\left\{m\bar{a}-\bar{f}(\tau)\right\}\exp\left(\frac{t-\tau}{\tau_1}\right)\right)d\tau=\left\{m\bar{a}-\bar{f}(\tau)\right\}\exp\left(\frac{t-\tau}{\tau_1}\right)\Bigg|_t^{\infty}=\bar{f}(t)-m\bar{a}=\frac{m\beta}{3}\int_t^{\infty}\frac{d\bar{a}}{d\tau}\exp\left(\frac{t-\tau}{\tau_1}\right)d\tau\Rightarrow$$

$$\bar{f}(t)=m\bar{a}+\frac{m\beta}{3}\int_t^{\infty}\frac{d\bar{a}}{d\tau}\exp\left(\frac{t-\tau}{\tau_1}\right)d\tau \qquad (8.11)$$

If, for low velocities, we assume that the force, the mass and the acceleration are invariant with respect to Galilean transformations, then each of eq.s (8.6,8.7),separately, turn out not to be Galilean invariant. Note that the fundamental law of dynamics for a particle with variable mass would be also not a Galilean invariant law if it only includes the particle's mechanical mass; however, if the entire system is considered, including the split matter that moves with velocity $\boldsymbol{v_s}$ (5.13) becomes

$$\bar{F}=\left(\bar{v}-\bar{v}_s\right)\frac{dm}{dt}+m\frac{d\bar{v}}{dt} \qquad (8.12),$$

which is the correct form for the fundamental law of dynamics and a Galilean invariant law due to the appearance of a relative velocity $\boldsymbol{v\text{-}v_s}$. The same occurs with eq. (8.11) or eq.s (8.6,8.7) as a system, which is a Galilean invariant law.

## 9. FLUCTUATIONS.

As shown, eq. 8.11 is contradictory. We see that the corresponding integral times $\tau$ extends to greater than the observation time $t$, which would be a problem in order to maintain the principle of causality in which this work is based. On the other hand the integral value is determined, at a time $dt$, from the equation; assuming known mass $m$, the force $\boldsymbol{f}$ and acceleration $\boldsymbol{a}$ at that instant. However the characteristic time $\tau_1$ of eq. 8.11 equals to $4\hbar/mc^2$ , into the quantum physics domain ; and we must abandon the idea of knowing accurately the relation $\boldsymbol{a}(t)$, $\boldsymbol{f}(t)$ at this time scale. Based on the relationships of energy-time uncertainty of quantum physics, the concept of *present* can not be reduced to a geometric point in the coordinate timeline, but we must accept an amplitude or uncertainty in this concept. In this context we can interpret that the limits of integration in 8.11 correspond to this amplitude over time. On time scales far above $\hbar/mc^2$ seconds have a net fluctuation of acceleration: $\Delta\boldsymbol{a}$, and linear dynamic eq. 8.11 combines the external force, the average acceleration and the fluctuation

$$\bar{f}(t) = m\bar{a} + m\Delta\bar{a} \quad ; \quad \Delta\bar{a} \cong \frac{\beta}{3}\int_t^{\infty}\frac{d\bar{a}}{d\tau}\exp\left(\frac{t-\tau}{\tau_1}\right)d\tau \quad (9.1)$$

Now we review the development done from the beginning. The hypothesis that radiation comes from a particle's kinetic energy leads us to consider the fluctuation of the mass of a radiating particle. The particle loses and regains mass. If the mass is lost at a rate that is higher than the recovery rate, then the difference is related to the kinetic energy that contributes to the radiation. However, if the rate of mass loss is different than the recovery rate, then the process cannot be continuous in time because the particle's existence would be compromised if it were. Therefore, the radiation emission process must fluctuate; that is, it must be discontinuous in time for the particle to have a margin of recovery. In regard to the radiation, the classical model of a point charge assumes that the particle has infinite electromagnetic energy and so infinite mass. In this model, a continuous loss of mass should not be a problem because the mass of the particle remains infinite; however, there is an evident mechanical contradiction because an infinite mass cannot be accelerated by a finite external force. In contrast, we have the Lorentz force, in which the particle's mechanical mass is clearly finite. For consistency, we must study the case in which the radiation is emitted by a particle of finite mass, and this leads us to consider a discontinuous radiation process when the charged particle must maintain a stable mass. Under these conditions, the Lorentz force and Larmor´s formula can only be statistical averages in reality. For both the kinetic energy and the radiated energy of a particle with constant mass, there is a relationship with the mechanical impulse; however, a direct transfer from the kinetic energy and impulse to radiation is not possible, as can easily be observed in the following reductio ad absurdum where the subscript *1* refers to a particle of constant mass and the subscript *2* refers to the radiation.

$$\Delta E_1 = \bar{v}\bullet\Delta\bar{P}_1 \; ; \; \Delta\bar{P}_2 = \frac{\bar{v}}{c^2}\Delta E_2 \; ; \Delta E_1 = -\Delta E_1 \; , \; \Delta P_2 = -\Delta P_2 \Rightarrow \Delta E_1 = \frac{v^2}{c^2}\Delta E_1 \qquad (9.2)$$

According to our hypothesis, the problem is that the radiation is associated with a fluctuation in the particle's mass that, on average, must keep the particle mass constant. When we introduce a small fluctuation of the mass, $\delta m$, into the relativistic energy $U$ , we obtain

$$U^2 = p^2c^2 + (m+\delta m)^2c^4 \approx p^2c^2 + \left(mc^2\right)^2 + 2m\delta mc^4 \Rightarrow \left(U - mc^2\right)\left(U + mc^2\right) \approx p^2c^2 + 2m\delta mc^4 \quad (9.3).$$

For velocities that are much lower than the speed of light, it is valid to suppose $U+mc^2 \approx 2mc^2$, and therefore,$U-mc^2 \approx p^2/2m+$□$mc^2$; if we compare this result with eq. (5.19), we obtain $qEx=U-mc^2$, and for the mass fluctuation,

$$c^2\delta m \approx \frac{q^2}{6\pi\varepsilon_0 c^3}a^2\delta t \quad (9.4).$$

Evidently, this mass fluctuation, which also appears in eq. (8.1), grows over time, and if it were not emitted in the form of radiation, it would necessarily affect the particle's stability. Therefore, the dynamics of the particle includes phases in which it accumulates mass and phases in which it releases the excess mass as radiation. Equation (8.9) indicates an effective increase of mass and then $\tau_1$ is a characteristic time of the accumulation phase. Equation (2.7) and Larmor´s formula indicates that $\tau_0$ is a characteristic time of the release phase. In a constant electric field, the accumulation phases provoke decreases in the acceleration by increasing the mass, and the release phases provoke increases in the acceleration by decreasing the mass. Thus the mass fluctuation entails a fluctuation of acceleration. If the accumulation phases are longer than the release phases on average, then the average effect of the process is similar to that of a friction force that tends to decrease the particle's velocity and acceleration. Equation (7.1) provides a way of measuring this phenomenon by relating variations in the acceleration to the amount of radiation emitted. In this context, eq. (5.19) must be interpreted as an average over the appropriate time interval. In this way, we can understand the apparent discrepancy between eq.s (8.1) and (8.11) when the acceleration is constant : *If there are no variations in the acceleration, then there are no mass fluctuations, and no radiation is emitted.* This outlook is stable if $\tau_1 >> \tau_0$ ; in appendix 12.5 we can see a limit in this inequality and a possible relation with phenomena in Solar Corona.
The concept of mass fluctuation is similar to what some authors call *Schott energy* or *acceleracion energy:*

*"Changes in the acceleration energy correspond to a reversible form of emission or absorption of field energy, which never gets very far from the electron."*[9]

## 10. Radiation And Acceleration In A Gravitational Field.

In the last section we found that If there are no variations in the acceleration, then there are no mass fluctuations, and no radiation is emitted. If the particle's acceleration is entirely due to a *gravitational field*, then fluctuations in its mass cannot affect its acceleration. This follows from the *equivalence principle of general relativity*, and therefore, a charge that is accelerated by a gravitational field does not experience the friction due to radiation; that is, the radiation will not cause variations in the particle's acceleration. Therefore, for an accelerating charged particle, we draw the following conclusions:

1-If the acceleration is due to a gravitational field, then the charge does not emit radiation.

2-The charge emits radiation only if its acceleration is due to an external electromagnetic field or, in general, a force that acts on the charge.

Equation (5.19) relates the kinetic energy and the radiated energy, but we have seen that there is no *lineal direct physical relationship* between these two energy forms. We can think that the average needed to arrive at eq. (5.19) goes through a *non-linear and irreversible* process. One sign of this non-linear process is that eq. (7.1) contains Planck's constant and eq. (5.19) does not. This non-linear process confronts the classical idea of energy as the displacement of forces and is related to the principle of causality, as we saw in the section on the dynamics of an accelerated charge.

There is a well-known image in which an observer inside a box accelerated by an external motor, assumed to be unobservable , is unable to distinguish between the box's real physical state and a state of rest in a uniform gravitational field. The equivalence principle, which is sometimes explained using this image, assumes a local physical symmetry between a gravitational observer and a different accelerated observer.

Therefore, we can imagine, for example, that by experimenting, the observer arrives at the following physical law: for any physical object to remain unaccelerated, a force of magnitude ***mg***, where ***m*** is the object's *inertial mass* and ***g*** is the inertial acceleration, must act on it. Invoking the symmetry of the equivalence principle shows that this is also true for an observer at rest on the earth's surface when ***g*** is interpreted as the intensity of the gravitational field and ***m*** is interpreted as the mass or *gravitational load*. The relevance of the symmetry is that the inertial mass and the gravitational mass are always equal to each other, which has been accepted without explanation since Newton's time. Therefore, the acceleration due to gravity is the same *regardless of the mass and the physico-chemical nature of the object;* these facts have been known since Galileo's time.

However, there are physical phenomena that seem not to obey the symmetry between gravity and acceleration. One in particular is the radiation emitted by an accelerated charge, which is known in classical electromagnetic theory. In this case, we begin with a charged object that is at rest. According to the theory the charge in the accelerated box emits radiation. However, the symmetry leads us to suppose that a charged object that is at rest on the earth's surface emits radiation. It seems that this does not occur, based on our experience on earth and knowledge obtained from astronomical data. Also, the origin of the radiated energy should be justified. The gravitational observer clearly does not appreciate the origin of this possible energy, and the accelerated observer would conclude that the source or origin of the radiated energy is not inside the box; instead, it comes from the external object that causes the box to accelerate. That is, this observer would give a *non-local* explanation for the radiation process.

The symmetry that we refer to is restricted to constant accelerations and gravitational fields. If we apply the dynamic equation, eq. (7.1), to a charge that moves with constant acceleration $\boldsymbol{a_0}$ in an external electric field, ***E***, we obtain

$$\frac{d}{dt}\left(\bar{a}_0 - \frac{q}{m}\bar{E}\right) = -\frac{\beta}{c^2}a_0^2\bar{v} \Rightarrow d\bar{E} = \frac{m}{q}\frac{\beta}{c^2}a_0^2\, d\bar{r} \Rightarrow \bar{E} = \bar{E}_0 + \frac{m}{q}\frac{\beta}{c^2}a_0^2\bar{r} \qquad (10.1)\cdot$$

In contrast, if we apply the equation of the conservation of energy, eq. (5.19), to the field, we obtain

$$\left(m\bar{a}_0 - q\bar{E}_0\right)\bullet\bar{v} - m\frac{\beta}{c^2}a_0^2\,\bar{r}\bullet\bar{v} + m\tau_0 a_0^2 = 0 \qquad (10.2)\cdot$$

When we derive this expression twice, we see that the result only makes sense if the acceleration,$\boldsymbol{a_0}$, is zero, which indicates that *there are no charges that move with constant acceleration due to the effect of an electric field.*

Imagine a classical pendulum with a charge on its end oscillating under the effects of string tension and gravity. According to the quantum theory of harmonic oscillators, states that cancel out the emitted radiation are possible. The tension is related to the chemical bonds that are between the molecules of two immediately adjacent parts of the string, and therefore, it is a force of *quantum nature* rather than an electromagnetic force.

These ideas allow us to think that there is no radiation emitted for an accelerated observer or for a gravitational observer, and thus, the symmetry is maintained.

## 11. CONCLUSIONS.

The classical electromagnetism has not correctly included the role of mechanical mass; and this is the origin of the problem on the radiation of an accelerated charge. In electromagnetism you can easily describe a point charge in arbitrary motion; but a point charge, according with Einstein´s mass-energy equivalence, has an infinite mass, and therefore can not be accelerated. A movement that is arbitrary and non-accelerated is absurd. A charge distribution is unstable if only electrical forces act and the best example is nuclear forces, but what is the corresponding force on the electron? This paper suggests that the mechanical mass of a charged particle fluctuates, so they are acting both tendencies: instability and recovery in a compensatory process. This leads us to the acceleration in Larmor's formula should be relative to an external electric and/or magnetic system. We have seen that this compensation is a non-linear process that depends on Planck's constant and, in this sense, it is of quantum nature. However we have also seen that electromagnetism is not totally alien to this compensatory mechanism and can describe at the same time a stable mass of the particle and physical causality. A sign of this aspect can be seen in *Lenz's law*: the currents associated with induced electromotive forces in a conductor by altering the external magnetic flux, generate magnetic fields which, in turn, tend to cancel external magnetic flux disturbance; and therefore the electromagnetic system reacts trying to restore equilibrium conditions. The stability we have found is similar to thermodynamic equilibrium because of the competition between opposing tendencies; this balance is conditional and may change or break for charges and accelerations sufficiently high, conditions that can happen in the solar corona.

# 12. APPENDIX.

## 12.1. Constant Electric Field

Equation (7.1) can be represented using the intrinsic coordinate system associated with the particle's trajectory. If the unit vectors are ***T***(tangent), ***N***(normal) and ***B***(binormal), then applying the *Frenet formulas* results in

$$\frac{d}{dt}\left(\frac{dv}{dt}\overline{T}+\frac{v^2}{\rho}\overline{N}\right)+\beta\left(\frac{a}{c}\right)^2 v\overline{T}=0 \quad \Rightarrow \frac{d^2v}{dt^2}\overline{T}+\frac{dv}{dt}\frac{v}{\rho}\overline{N}+\frac{d}{dt}\left(\frac{v^2}{\rho}\right)\overline{N}+\frac{v^3}{\rho}\left(-\frac{1}{\rho}\overline{T}+\tau\overline{B}\right)=-\beta\left(\frac{a}{c}\right)^2 v\overline{T} \qquad (12.1.1),$$

where $\rho$ is the curvature and $\tau$ is here the torsion of trajectory. The equality of the vectors requires the torsion to be null, and therefore, the particle's trajectory is planar. In the component ***N***,

$$\frac{dv}{dt}\frac{v}{\rho}+\frac{d}{dt}\left(\frac{v^2}{\rho}\right)=0 \Rightarrow \rho = kv^3 \qquad (12.1.2),$$

where $k$ is a constant; therefore, the curvature is proportional to the cube of the velocity. Combining this result with eq. (5.16) and using the acceleration components, we obtain

$$\left(\frac{dv}{dt}\right)^2 = a^2 - \left(\frac{v^2}{\rho}\right)^2 = A^2\exp\left(-\beta v^2/c^2\right)-\frac{1}{k^2v^2} \Rightarrow \left[kv\left(\frac{dv}{dt}\right)\right]^2 = A^2k^2v^2\exp\left(-\beta v^2/c^2\right)-1 \Rightarrow \frac{k}{2}\frac{dv^2}{dt} = \pm\sqrt{A^2k^2v^2\exp\left(-\beta v^2/c^2\right)-1} \Rightarrow$$

$$\frac{dv^2}{dt} = \pm\frac{2}{k}\sqrt{A^2k^2v^2\exp\left(-\beta v^2/c^2\right)-1} \qquad (12.1.3)$$

This expression is in principle directly integrable, and $v^2(t)$ can be obtained. As a result, we see that there is a case in which the velocity decreases with time, as it should for a particle with an initial velocity that opposes the electric field, and cases in which the velocity increases with time. As we see in *Fig. 1* there is a lower bound on the velocity module of approximately *1/kA*. In this way, the solution explicitly eliminates velocities of zero, which is consistent with the problem about kinetic energy discussed in eq. (5.19).

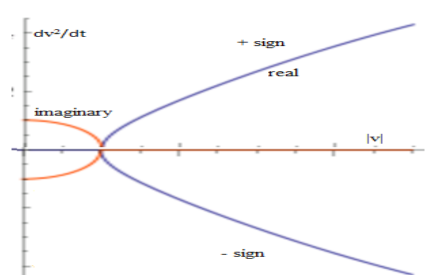


*Fig. 1 :*$dv^2$/dt versus *velocity modulus*

Note the reader that zero velocity in this case would mean repose regarding the fountains of the external electrostatic field.

## 12.2. Constant Magnetic Field.

Using the equation of energy, eq.(5.19),and the result given in eq.(5.16), we obtain

$$\frac{dE_c}{dt}+m\tau_0 a^2=0\;;\;a=a_0\exp\left(-\frac{\beta}{2c^2}\left(v^2-v_0^2\right)\right)\Rightarrow\frac{1}{2}m\frac{dv^2}{dt}=-m\tau_0 a_0^2\exp\left(-\frac{\beta}{c^2}\left(v^2-v_0^2\right)\right)\Rightarrow\int_{v0}^{v}\exp\left(\frac{\beta}{c^2}\left(v^2-v_0^2\right)\right)d(v^2)=-2\tau_0 a_0^2 t\Rightarrow$$

$$\left.\frac{c^2}{\beta}\exp\left(\frac{\beta}{c^2}\left(v^2-v_0^2\right)\right)\right|_{v0}^{v}=-2\tau_0 a_0^2 t\Rightarrow\exp\left(\frac{\beta}{c^2}\left(v^2-v_0^2\right)\right)-1=-2\frac{\beta}{c^2}\tau_0 a_0^2 t\Rightarrow$$

$$v^2=v_0^2+\frac{c^2}{\beta}\ln\left(1-2\frac{\beta}{c^2}\tau_0 a_0^2 t\right);\;\tau_0=\frac{q^2}{6\pi\varepsilon_0 c^3 m}\qquad(12.2.1)$$

under normal conditions the argument of the logarithm is very close to *1* and can approximate

$$v^2\approx v_0^2-2\tau_0 a_0^2 t\qquad(12.2.2)$$

## 12.3. Deflection Of Charged Particle By Coulomb Centre Of Force : Bremsstrahlung.

Rutherford´s famous experiment on atomic structure was based on the deflection of charged particles by centres of force associated with the atomic nuclei in an ultrathin layer of gold. Evidently, the acceleration generated by the centre of force on the particle causes radiation to be emitted, which should produce an increase in the deflection of a trajectory that we can consider approximately hyperbolic, in accord with the Newtonian formalism of forces. If we assume that the accelerated particle's motion is planar, we can use polar coordinates in the complex plane to write eq. (7.1) as follows:

$$v=\left(\frac{dr}{dt}+ir\frac{d\theta}{dt}\right)e^{i\theta}\;;\;a=\left[\frac{d^2r}{dt^2}-r\left(\frac{d\theta}{dt}\right)^2+i\left(2\frac{dr}{dt}\frac{d\theta}{dt}+r\frac{d^2\theta}{dt^2}\right)\right]e^{i\theta}\quad;\quad\frac{f}{m}=\frac{A}{r^2}e^{i\theta}\quad;\quad\frac{d}{dt}\left(a-\frac{f}{m}\right)=-\frac{\beta}{c^2}a^2 v\Rightarrow$$

$$\frac{d^3r}{dt^3}-3\frac{dr}{dt}\left(\frac{d\theta}{dt}\right)^2-3r\frac{d\theta}{dt}\frac{d^2\theta}{dt^2}+2\frac{A}{r^3}\frac{dr}{dt}+i\left(3\frac{dr}{dt}\frac{d^2\theta}{dt^2}+r\frac{d^3\theta}{dt^3}+3\frac{d\theta}{dt}\frac{d^2r}{dt^2}-r\left(\frac{d\theta}{dt}\right)^3-\frac{A}{r^2}\frac{d\theta}{dt}\right)=-\frac{\beta}{c^2}a^2\left(\frac{dr}{dt}+ir\frac{d\theta}{dt}\right)\qquad(12.3.1)$$

Let us take the real component of the previous equation and approximate a constant angular momentum, as in the Newtonian model, as follows:

$$\frac{d^3r}{dt^3}-3\frac{dr}{dt}\left(\frac{d\theta}{dt}\right)^2-\frac{3}{2}r\frac{d}{dt}\left(\frac{d\theta}{dt}\right)^2+2\frac{A}{r^3}\frac{dr}{dt}+\frac{\beta}{c^2}\frac{A^2}{r^4}\frac{dr}{dt}=0;\;a\approx\frac{f}{m}=\frac{A}{r^2}e^{i\theta};\;L=mr^2\frac{d\theta}{dt}\qquad(12.3.2),$$

where we also approximate the square of the acceleration in the radiation term using its Newtonian value. Removing the derivative of the angle from the angular momentum ***L***, we obtain

$$\frac{d^3r}{dt^3}+\left(\frac{3L^2}{m^2}-\frac{\beta A^2}{c^2}\right)r^{-4}\frac{dr}{dt}+2Ar^{-3}\frac{dr}{dt}=0\Rightarrow\frac{d}{dt}\left(\frac{d^2r}{dt^2}-\frac{1}{3}\left(\frac{3L^2}{m^2}-\frac{\beta A^2}{c^2}\right)r^{-3}-Ar^{-2}\right)=0\qquad(12.3.3).$$

Applying the initial conditions for $r$ very far from the centre of force and multiplying by $dr/dt$, we continue the integration process as follows:

$$\frac{d^2r}{dt^2}-\frac{1}{3}\left(\frac{3L^2}{m^2}-\frac{\beta A^2}{c^2}\right)r^{-3}-Ar^{-2}=0\;\Rightarrow\;\frac{d^2r}{dt^2}\frac{dr}{dt}-\frac{1}{3}\left(\frac{3L^2}{m^2}-\frac{\beta A^2}{c^2}\right)r^{-3}\frac{dr}{dt}-Ar^{-2}\frac{dr}{dt}=0\Rightarrow\frac{d}{dt}\left(\frac{1}{2}\left(\frac{dr}{dt}\right)^2+\frac{1}{2}\left(\frac{L^2}{m^2}-\frac{\beta A^2}{3c^2}\right)r^{-2}+Ar^{-1}\right)=0\Rightarrow$$

$$\left(\frac{dr}{dt}\right)^2+\left(\frac{L^2}{m^2}-\frac{\beta A^2}{3c^2}\right)r^{-2}+2Ar^{-1}=2B \qquad (12.3.4)$$

Removing the variable *dt* from the angular momentum, ***L***, we obtain

$$\left(\frac{dr}{d\theta}\frac{L}{mr^2}\right)^2+\left(\frac{L^2}{m^2}-\frac{\beta A^2}{3c^2}\right)r^{-2}+2Ar^{-1}=2B\Rightarrow\frac{L^2}{2m}\left[\frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2+\left(1-\frac{\beta A^2m^2}{3c^2L^2}\right)\frac{1}{r^2}\right]+\frac{Am}{r}=Bm \qquad (12.3.5).$$

The reader can check the similarity of this result with the Newtonian case in [6]; it allows us to write the following after solving for *A* and *β* and relating *B* to the energy *E:*

$$\frac{L^2}{2m}\left[\frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2+\left(1-\frac{\alpha^3Z_m^4Z_r^2}{3}\left(\frac{\hbar}{L}\right)^2\right)\frac{1}{r^2}\right]\pm\frac{Z_mZ_re^2}{4\pi\varepsilon_0 r}=E \qquad (12.3.6),$$

where $\alpha$ is the fine-structure constant, $\hbar$ is Planck's constant, *Z* is the number of elementary charges, the subscript *m* refers to the moving charge and the subscript *r* refers to the charge that is at rest. The positive sign represents repulsion and the negative sign represents attraction between the particle and the centre of force. As in [6], we find the corresponding *Binet's* differential equation, which is

$$\frac{d^2u}{d\theta^2}+\left(1-\frac{\alpha^3Z_m^4Z_r^2}{3}\left(\frac{\hbar}{L}\right)^2\right)u\pm\frac{Z_mZ_re^2m}{4\pi\varepsilon_0L^2}=0\;;\;u=\frac{1}{r} \qquad (12.3.7),$$

and the corresponding solution for the trajectory is

$$\frac{1}{r}=\left(\sqrt{\frac{2mE}{L^2}+\left(\frac{Z_mZ_re^2m}{4\pi\varepsilon_0L^2}\right)^2}\right)\cos\left(\theta\sqrt{1-\frac{\alpha^3Z_m^4Z_r^2}{3}\left(\frac{\hbar}{L}\right)^2}\right)\mp\frac{Z_mZ_re^2m}{4\pi\varepsilon_0L^2} \qquad (12.3.8),$$

with a negative sign for repulsion and a positive sign for attraction between the centre of force and the particle. If we call the trajectory's Newtonian deflection $\Delta_N$, we see right away that the effect of the radiation is an increase in the deflection,$\Delta$,

$$\tan(\Delta_N/2)=\frac{Z_mZ_re^2m}{LP_\infty};\quad \Delta\sqrt{1-\frac{\alpha^3Z_m^4Z_r^2}{3}\left(\frac{\hbar}{L}\right)^2}=\Delta_N\;\Rightarrow\;\Delta=\frac{\Delta_N}{\sqrt{1-\frac{\alpha^3Z_m^4Z_r^2}{3}\left(\frac{\hbar}{L}\right)^2}} \qquad (12.3.9),$$

where $P_\infty$ is the modulus of the linear momentum very far from the centre of force. According to this result, the corrective term is only appreciable in practice when the angular momentum, *L*, is on the order of Planck's constant,

$$L=bmv\approx\hbar\approx\Delta x\Delta p \qquad (12.3.10),$$

where *b* is the *straight arm* of the angular momentum or the distance between the centre of force and the asymptote. The previous relationship indicates that the moving particle passes very close to the "fixed" centre of force, at a distance comparable to the *De Broglie* wavelength of the moving particle. Therefore, at this level, the phenomenon must be studied using quantum mechanics as an individual *photon emission* process. Tipical data on the alpha particles in Rutherford's experiment are as follows: mass = $6.64424\times10^{-27}$ kg and initial velocity = $2\times10^{7}$ m/s. These data lead to $b\approx8\times10^{-16}$ metres. In the case of an alpha particle aimed directly towards a gold nucleus (which, therefore, has ***L****=0*), the maximum approach, *r*, can be calculated using the conservation of energy; the result is

$$\frac{1}{2}mv^2=\frac{Z_mZ_re^2}{4\pi\varepsilon_0r} \qquad (12.3.11),$$

which assumes that $r\approx2.7\times10^{-14}$ metres, which is very far from the previous value of *b*; therefore, the corrections presented here are negligible in the case of Rutherford's experiment.

## 12.4. Simple Oscillator.

For a charged particle oscillating along the *x axis* subjected to a force $F=-kx$, we can present eq. (7.1) as

$$\frac{d}{dt}\left(a+\omega^2 x\right)=-\frac{\beta}{c^2}a^2 v;\quad \omega^2=\frac{k}{m}\qquad (12.4.1)$$

This equation can be easily integrated as follows:

$$\frac{da}{dt}+\omega^2 v=-\frac{\beta}{c^2}a^2 v\Rightarrow\int_0^a\frac{da}{1+\left(\frac{\sqrt{\beta}}{\omega c}a\right)^2}=-\int_{x0}^{x}\omega^2 dx\Rightarrow\frac{\omega c}{\sqrt{\beta}}\arctan\left(\frac{\sqrt{\beta}}{\omega c}a\right)=-\omega^2(x-x_0)\Rightarrow$$

$$a=-\frac{\omega c}{\sqrt{\beta}}\tan\left(\frac{\omega\sqrt{\beta}}{c}(x-x_0)\right)\quad (12.4.2)$$

where $x_0$ is a point of null acceleration for the particle. We can see the physical meaning of $x_0$ by *Fig. 2*, that depicts two Cartesian axes *x-y* and a charged particle moving to the right (dashed arrow) on the *x* axis. The two arms of the particle radiation are also depicted; and, with continuous arrow, the forces acting on the particle. The solid arrow to the right corresponds to the force $F = -kx$. In the path of the particle between the ends $[-A, A]$ will be an average reaction force (arrow to the left) associated with radiation in a half cycle. For small values of the argument of the tangent function in (12.4.2) we have $-\omega^2 x(t)+\omega^2 x_0 \approx a$ so that $x_0$ appears

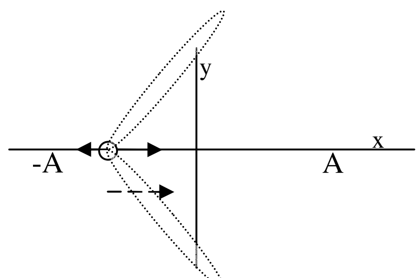


*Fig. 2 :A point charge oscilating*

related to the average radiation force in a half cycle. This point $x_0$ also affects energy as equation (5.19) tell us in this case

$$\left(\frac{\omega c}{\sqrt{\beta}}\tan\left(\frac{\omega\sqrt{\beta}}{c}(x-x_0)\right)-\omega^2 x\right)v=\tau_0 a^2\qquad (12.4.3)$$

and to a first approximation for a small argument of the tangent, we have

$$\omega^2 x_0 v\approx-\tau_0 a^2\quad (12.4.4)$$

This equation can not be verified, instantly, for a classical oscillatory movement (low radiation), as it does not reflect that an increase in the acceleration module entails a fall in module speed. Assuming valid eq. 12.4.4 *on average*, the *energy conservation* needs $x_0 v$ product is always negative. Changing the sign of the velocity at the end of each half cycle involves a change in the position $x_0$ to either side of the origin of coordinates and (12.4.4) qualitatively predicts the behaviour shown in the drawing. Therefore the value $x_0$ must be considered variable over time: $x_0(t)$, similar to a square wave and the average eqs. (12.4.2;12.4.4) that mentioned above refers to each half cycle where both, speed direction and point $x_0$, are held constant. In the half-cycle change we must assume a discontinuity, changing the initial condition $x_0$.

We can calculate the dynamic equation using the following approaches:

1-Simple harmonic motion of the particle:

$$x \approx Asen(\omega t)\ ;\ v \approx \omega A\cos(\omega t); a \approx -\omega^2 Asen(\omega t)\ ;\ \frac{da}{dt} \approx -\omega^3 A\cos(\omega t) \quad (12.4.5)$$

2-Simple harmonic motion of point $x_0$ and average value of eq. (12.4.4)

$$x_0 \approx -A_0\cos(\omega t) \quad ; \quad A_0 \approx \tau_0 \omega A \qquad (12.4.6)$$

this last approach is equivalent to the first component of Fourier series corresponding to the square wave $x_0(t)$. We develop the dynamic equation as follows

$$Dynamic\ eq.: -\omega^2 x(t) + \omega^2 x_0(t) \approx a$$

$$\omega^2 x_0(t) \approx -\omega^2 \tau_0 \omega A\cos(\omega t) \ \approx \tau_0 \frac{da}{dt} \approx -\omega^2 \tau_0 v \Rightarrow \begin{cases} a - \tau_0 \dfrac{da}{dt} + \omega^2 x \approx 0 & (12.4.7) \\ a + \omega^2 \tau_0 v + \omega^2 x \approx 0 & (12.4.8) \end{cases}$$

these two equations are equivalent in our approach and we can see for the first (12.4.7) the shape of the classical equation (2.8).

We can imagine the general motion of a charge affected by a force as a set of half cycles with change of initial conditions at the ends. In each half cycle conservation of momentum and energy is verified on average. If the time duration of these half cycles is small enough, at a microscopic scale, the particle appears to move by shocks, although on average according to the direction and intensity established by the external force. In my opinion Heisenberg's relations reflect the way nature creates these half cycles.

## 12.5. Solar Corona.

The corona is the outermost layer of the solar atmosphere; it contains intense magnetic fields and plasmas that contain highly ionized atoms, such as iron atoms that lack 19 electrons [7]. The radiation emitted from the corona generally has high frequencies; ultraviolet light and X-rays predominate in its spectrum. All this leads researchers to assign a temperature of millions of degrees to the solar corona, which leaves us perplexed if we recall that the temperature of the sun's surface is approximately 5.000 celsius. One would expect a progressive decrease in the temperature as the distance increases, as is the case with a bonfire.

In our approach, we can see expression $\tau_1=6\tau_0/\beta$ (see eq. 8.11) as the relation among time of accumulation phase ($\tau_1$) and release phase ($\tau_0$) and it should be $\tau_1>>\tau_0$; inequality that has a clear limit

$$\tau_1 >> \tau_0 \Rightarrow \frac{6}{\beta} = \frac{6}{Z^2\alpha} >> 1 \Rightarrow Z << \sqrt{\frac{6}{\alpha}} \approx \sqrt{6\times 137} \approx 28 \quad (12.5.1)$$

the presence of highly charged ions means a value of Z elevated, and this may affect to the time balance among the phases in the radiation dynamic. Also, the presence of an intense magnetic field and magnetic reconnection phenomena[8] favour an increase in acceleration and then radiation braking. The emitted radiation can also increase the ionization of the particle or of other particles from the inner layers of the solar atmosphere (*spicules*). In this way, the presence of intense magnetic fields in the corona can provoke a combination of high-frequency radiation and particle velocities that are relatively low for the expected temperature. The classical explanation of the *Fraunhofer lines* in the solar light spectrum implies that there are radiation-absorbing cold gases in the external layers of the sun.

## REFERENCES


[1] Landau L D and Lifshitz E M (1975) The Classical Theory of Fields (Oxford: Elsevier)
[2] Bredow M M, Rumyantsev V V and Toptygin I N (1985) Classical Electrodynamics (Moscow, Nauka).
[3] Feynman R, Leighton R and Sands M (1965) :The Feynman Lectures on Physics. Vol 2. Reading,MA,Adisson-Wesley
[4] Wangsness,R.K., (1979),Electromagnetic Fields, John Wiley and Sons.
[5] Purcell E. M. and Morin J. (2013), Electricity and Magnetism (Berkeley Physics Course, Vol. 2), Cambridge University Press.
[6] Goldstein H. (1980) Classical Mechanics (2nd ed.). Addison-wesley.
[7] About the Solar Corona: http://en.wikipedia.org/wiki/Corona, https://www.youtube.com/watch?v=Hg5tla7s-ys, https://www.youtube.com/watch?v=HFT7ATLQQx8
[8] About Magnetic Reconnection: nasa.gov May-12-2016: NASA Directly Observes Fundamental Process of Nature for 1st Time Science Journal DOI: 10.1126/science.aaf9987
[9] Dirac, P. A. M. (1938).Classical theory of radiating electrons, Proc. Roy. Soc. London A167, 148


## AUTHOR


*Enrique Cantera del Río is* Graduate *in Physical Sciencies and Technical Engineer in Telecommunications by Valladolid University (Spain).*