

# TRANSIENT NUMERICAL ANALYSIS OF INDUCTION HEATING OF GRAPHITE CRUCIBLE AT DIFFERENT FREQUENCY

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## **Abstract**

*Mathematical modeling of Induction heating process is done by using 2D axisymmetric geometry. Induction heating is coupled field problem that includes electromagnetism and heat transfer. Mathematical modeling of electromagnetism and heat transfer is done by using maxwell equations and classical heat transfer equation respectively. Temperature dependent material properties are used for this analysis. This analysis includes coil voltage distribution, crucible electromagnetic power, and coil equivalent impedance at different frequency. Induction coil geometry effect on supply voltage is also analyzed. This analysis is useful for designing of induction coil for melting of nonferrous metal such as gold, silver, uranium etc.*

## **Keywords:**

*Induction heating, FEM, Coil design, Graphite*

## **1.INTRODUCTION**

Graphite has been widely utilized in different industries applications, because of its physical properties like good thermal stability, corrosion resistance, high electrical conductivity, thermal shock resistance, high melting temperature, high purity, refractoriness, machinability etc [1]. Graphite electrical, mechanical and thermal properties makes it suitable for induction melting of non ferrous materials such as gold, silver, uranium etc.

In induction heating, graphite crucible is coupled with pulsating magnetic field produced by induction coil, which generates the electro motive force and eddy current in graphite crucible and that will heat it by joules effect. This heat is transferred to the charge (Material that is supposed to melt) through conduction, convection and radiation [2] [3].

Induction heating is multiphysics phenomena i.e combination of electromagnetism and heat transfer [4]. These physics are nonlinearly coupled with each other due to temperature dependent material properties. Mathematical modeling of electromagnetism and heat transfer is done by well known maxwell equations and classical heat transfer equation respectively [2][4] [4]. Field equations are solved by using finite element method.

This paper presents mathematical modeling of induction heating of graphite crucible. Electromagnetic power induced in different wall thickness of graphite crucibles, and voltage distributions in induction coil are analyzed at different frequency. This model helps to design and optimized the induction coil and graphite crucible for heating application.

This paper is organized as follows, section II gives brief description of induction heating system. Mathematical modeling and numerical solution procedure are explained in section III. In section IV, numerical results and analysis are present. Finally conclusion is given in section V.

## 2.SYSTEM DESCRIPTION

Induction melting set up comprises of power source, water cooled copper induction coil, graphite crucible and charge. Power source supplies high frequency current to the induction coil to generate varying magnetic field and heat the crucible.

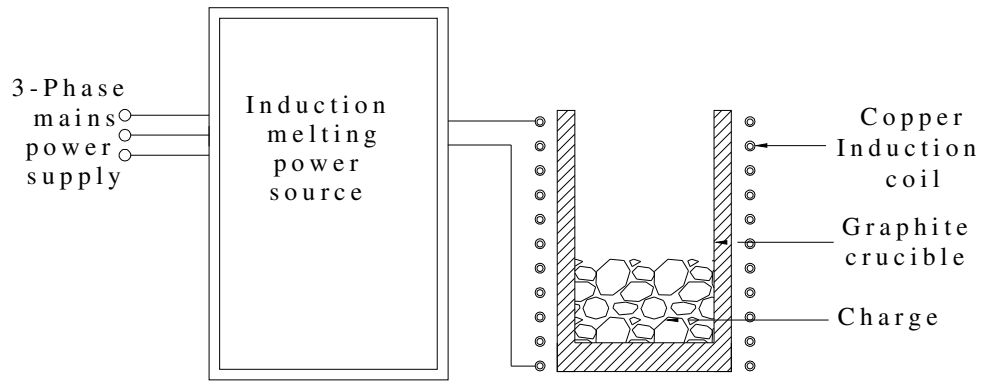


Figure 1:- Schematic of Induction heating system

## 3.MATHEMATICAL MODEL

Mathematical modeling of electromagnetism and heat transfer is done separately. Electromagnetic model is governed by maxwell equations as shown below [5] [6] [7] [8] [9],

$$\nabla \cdot B = 0 \quad (1)$$

$$\nabla \cdot D = \rho_c \quad (2)$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (3)$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} \quad (4)$$

Here, H: - Magnetic field strength (A/m)  
 E: - Electric field strength (V/m)  
 $\sigma$ : - Electrical conductivity (S/m)  
 J= Current density (A/m<sup>2</sup>)

D= Electric flux density(C/m<sup>2</sup>)  
 ρc=Electric charge density(C/m<sup>3</sup>)  
 B= Magnetic flux density (Wb/m<sup>2</sup>)

Constitutional equations for linear isotropic medium,

$$J = \sigma(T)E \quad (5)$$

$$B = \mu_0\mu_r H \quad (6)$$

$$D = \varepsilon_0\varepsilon_r E \quad (7)$$

Here,

$\mu_0$ = Free space magnetic permeability (H/m)  
 $\mu_r$ = Relative magnetic permeability  
 $\varepsilon_0$ = Free space electric permittivity (F/m)  
 $\varepsilon_r$ = Relative electric permittivity

Magnetic field problems are generally solved by using magnetic vector potential formulation and which is derived by using maxwell equations. Magnetic vector potential (A) is defined as,

$$B = \nabla \times A \quad (8)$$

From eq (3), (4) and (8), Magnetic vector potential equation in frequency domain can be written as,

$$\frac{1}{\mu_0\mu_r(T)}\nabla^2 A + J_s - j\omega\sigma(T)A = 0 \quad (9)$$

Here,

$J_s$ = source current density (A/m<sup>2</sup>)  
 $\omega$ =Angular frequency (rad/sec)

For solving eq (9) in axisymmetric geometry shown in figure 2, following assumption are considered,

1. The system is rotationally symmetric about Z-Axis.

2. All the materials are isotropic.
3. Displacement current is neglected.
4. Electromagnetic field quantities contents only single frequency component.

For different domain of figure (2), eq (9) can be written as,

$$\frac{1}{\mu_0\mu_r(T)}\nabla^2 A = 0 \quad \text{in } \Omega_1 \quad (10.1)$$

$$\frac{1}{\mu_0\mu_r(T)}\nabla^2 A + J_s - j\omega\sigma(T)A = 0 \quad \text{in } \Omega_2 \quad (10.2)$$

$$\frac{1}{\mu_0\mu_r(T)}\nabla^2 A - j\omega\sigma(T)A = 0 \quad \text{in } \Omega_3 \quad (10.3)$$

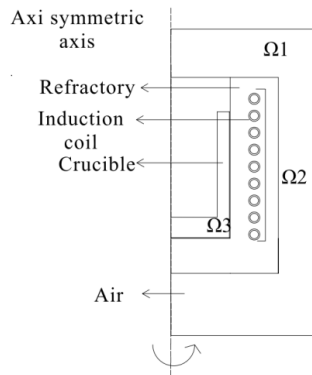


Figure 2:- 2-D Axisymmetric geometry of induction heating system

Eddy current and induced electromagnetics power in graphite crucible are calculated by using magnetic vector potential as shown below,

$$J_e = \sigma(T)(j\omega A) \quad (11.1)$$

$$Q = \frac{J_e^2}{\sigma(T)} = \sigma(T)(j\omega A)^2 \quad (11.2)$$

Here,

$J_e$ = induce eddy current density (A/m<sup>2</sup>)

Electromagnetism is coupled to heat transfer by power induced (Q) in graphite crucible and that is used as forcing function in heat transfer equation. Temperature in the graphite crucible is governed by the classical heat transfer equation [2] [5] [9] [10],

$$K(T) \cdot (\nabla^2 T) + Q = \rho c_p(T) \frac{\partial T}{\partial t} \quad (12)$$

Here,

- T= Temperature (DegK)
- $\rho$ = Density (Kg/m<sup>3</sup>)
- $c_p$ = Specific heat ((J/(Kg.K))
- K=Thermal conductivity (W/(m.K))
- $Q_{conv}$ = convection heat loss (W/m<sup>2</sup>)
- $Q_{rad}$ = radiation heat loss (W/m<sup>2</sup>)
- t =Time (Sec)

For electromagnetism, Dirichlet (A=0) and Neumann boundary conditions and for heat transfer, convection and radiation boundary conditions are used.

Convection heat loss can be represent as,

$$Q_{conv} = h \cdot (T - T_{amb}) \text{ W/m}^2 \quad (13)$$

Radiation heat loss can be represent as,

$$Q_{rad} = \epsilon \sigma_b \cdot (T^4 - T_{amb}^4) \text{ W/m}^2 \quad (14)$$

Here,

- h=convection coefficient (W/m<sup>2</sup>K)
- $\epsilon$ =Emissivity
- $\sigma_b$ = Boltzmann constant(5.67X10<sup>-8</sup> W/m<sup>2</sup>K<sup>4</sup>)
- $T_{amp}$ = Ambient temperature (DegK)

Induction heating is non linear coupled field problem that makes difficult to solve by analytical method. Hence, Numerical method is used to solve the field equations. Finite element method (FEM) is simple and most preferred numerical method for solving field equation, therefore, FEM is chosen for this analysis. FEM converts continuous equations i.e eq (10) into discrete equation by discretizing geometry of solution domain [2].Discretization can be done by using trigular, rectangular or hexagonal elements. Discretized equations are solved by using segregated or coupled field solver.

#### 4.SIMULATION

Simulation is done in three steps i.e. preprocessing, processing and post processing using FEM based multiphysics software [7].

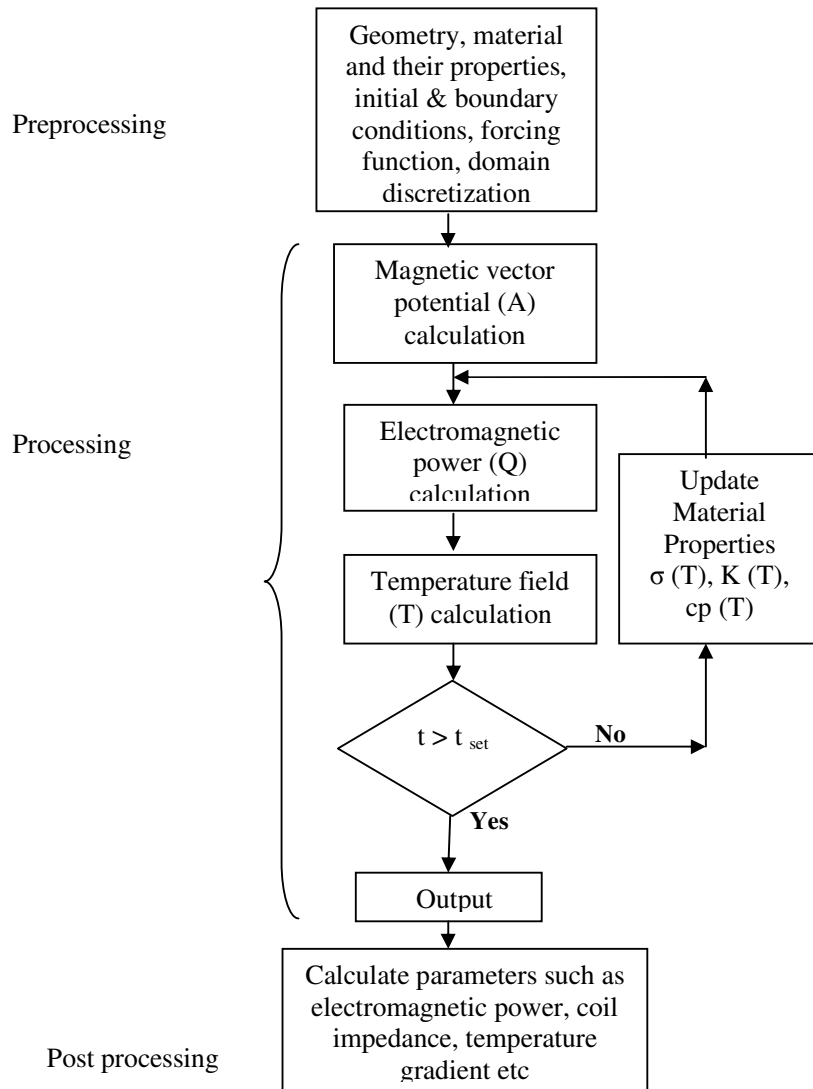


Figure 3. Computation steps

Preprocessing steps includes creating geometry, assigning materials and their properties to different sub domains, define boundary conditions, initial conditions, forcing function and domain discretization. Processing steps includes the computation of parameters as shown in figure 3. Post processing step includes computation of parameters such as coil impedance, convection and radiation losses etc for further analysis.

### 5. NUMERICAL RESULTS

Geometry shown in figure 4 (a) is used for simulation. Solution domain is discretized using trigular elements. Meshing density is more on Surface of graphite crucible and induction coil due to skin effect as shown in the figure 4(b). Graphite temperature dependent material properties are given in figure 4.

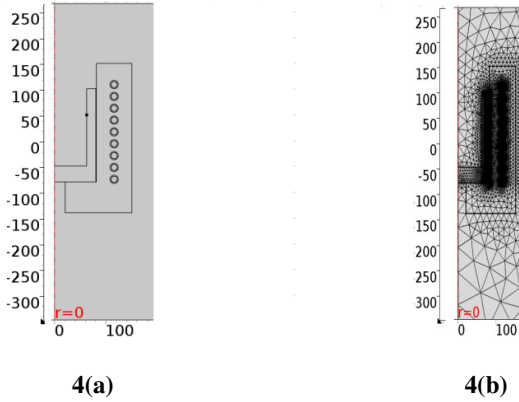
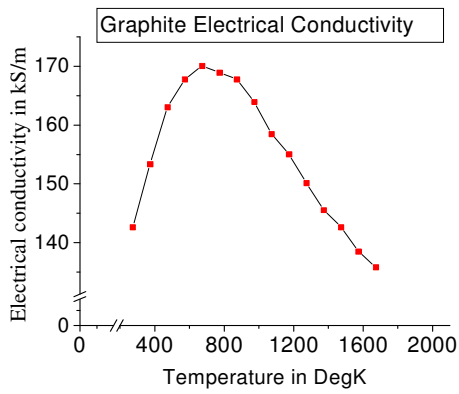
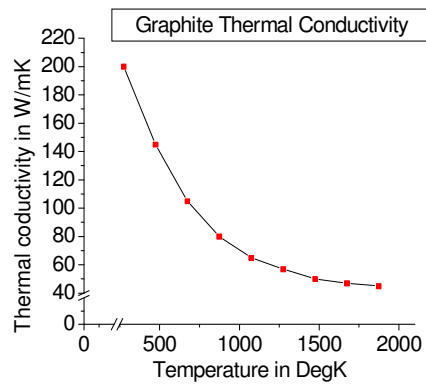


Figure 4(a). 2-D axisymmetric geometry, 4(b) Domain discretization (Meshing)

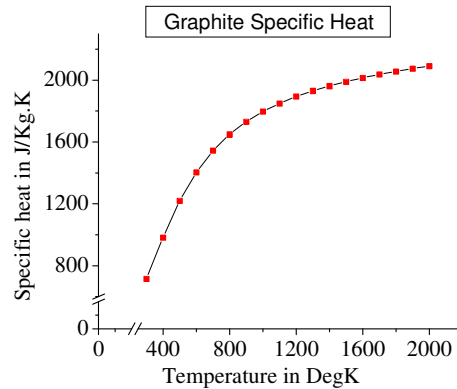
Dimension details of induction coil is given in table-I. Graphite temperature dependent material properties such as electrical conductivity, thermal conductivity, specific heat, are shown in figure 5.



5(a)



5 (b)



5 (c)

Figure 5:- Temperature dependent material properties of Graphite [2]

Table-I Induction coil dimensions and properties

Induction coil	Description
Material	Copper
Inside diameter	215
Outside diameter	247
Height	200
Coil tube diameter	16
Coil tube thickness	5
No. of turn	9

Initial conditions, boundary conditions and forcing function for electromagnetism and heat transfer are given in table-II and table-III respectively. Electromagnetism equations are solved in complete solution domain (induction coil, graphite crucible, air). Heat transfer analysis is done only in graphite crucible, because induction coil is water cooled, and always at room temperature.

Table-II Boundary condition and forcing function for electromagnetism

Boundary condition	Description
Outer boundary	$A=0$
Asymmetry axis	$\frac{\partial A}{\partial n} = 0$
Induction coil current	707.21A



Table-III Initial and boundary condition for Heat transfer

Boundary condition	Description
Initial temperature	303 DegK
Convection coefficient(h)	10 (W/m <sup>2</sup> K)
Emissivity( Graphite surface)	0.7
Emissivity(Refractory surface)	0.01

As induction coil carries single frequency current (because of series resonance configuration), Hence, electromagnetic analysis is done in frequency domain. Heat transfer equation is solved in transient domain. Magnetic field produced by induction coil at 1 kHz, 5 kHz and 9 kHz is shown in the figure 7.

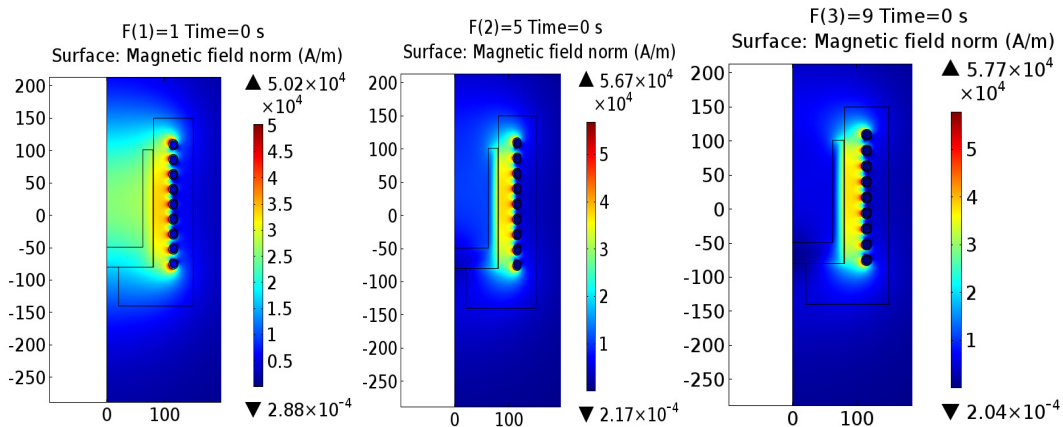


Figure 7. Magnetic field at 1 kHz, 5 kHz, and 9 kHz

## 6.RESULT AND ANALYSIS

From figure 7, it is observed that, magnetic field is high on graphite crucible surface and it reduces toward centre of the crucible. At 1 kHz, magnetic field penetrates more compared to 5 kHz and 9 kHz. Figure 8 shows the crucible temperature at different frequency, from figure, it is observe that as frequency increases crucible heat up at faster rate and that is because of increase in coupling between crucible and induction coil.

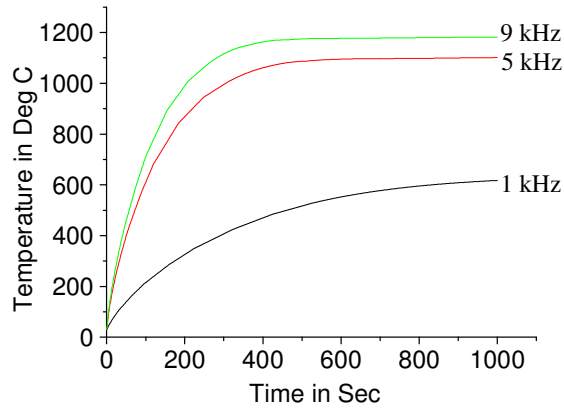


Figure 8. Graphite crucible temperature

Figure 9 shows, voltage distribution in different turn of the induction coil. From figure it is observed that from centre turn to outer turn of induction coil, impedance of coil turns are reduced and that is due to maximum proximity effect experience by centre turn compared to other turns. That makes higher voltage drop and power loss in centre turn of the induction coil. This analysis helps the designer for designing of efficient cooling system for induction coil.

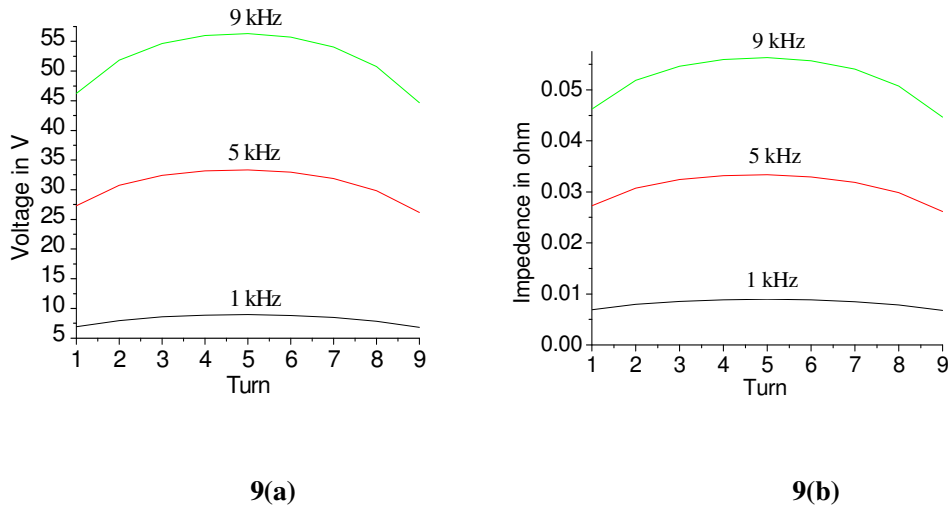


Figure 9. (a) Turn voltage, 9(b) Turn impedance

Figure 10 shows that electromagnetic power induced in graphite crucible at different frequencies. From figure, it is observed that during initial period, power induced in graphite crucible is first reduced and then increase and the reason for this is uniqueness of graphite electrical conductivity, which has both positive temperature coefficient and negative temperature coefficient as shown in figure 5(a).

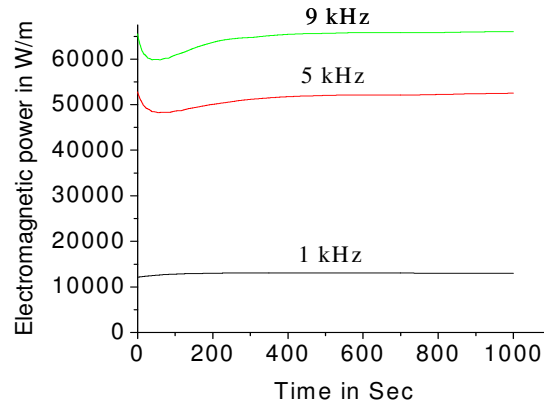


Figure 10. Electromagnetic power in crucible at different frequency

Figure 11 shows the variation of electromagnetic power in crucible at different thickness. From figure it is observed that as crucible thickness increase induce power is increases upto a level and after that it reduces. This is due to the skin effect, means 84%~ power is induced in one skin depth and 98 % ~ in second skin depth.

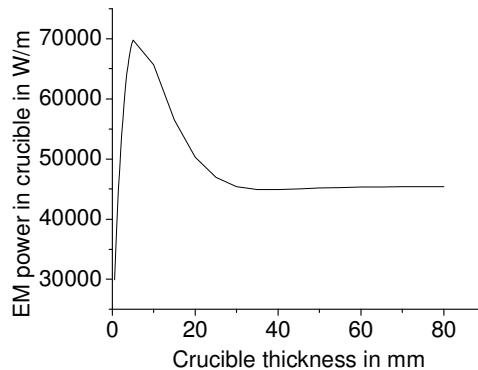


Figure 11. Electromagnetic power in crucible at different thickness

## 7. CONCLUSION

Transient numerical analysis of Induction Heating of Graphite Crucible was carried out successfully at different frequency. Electromagnetic power induced in graphite crucible is analyzed at different temperature that gives better understanding of heat transfer in graphite crucible. Voltage at each coil turns are analyzed, that helps in designing of cooling system for induction coil. Effect of crucible wall thickness on induced power is analyzed, that helps for selection of optimum crucible thickness.

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