MODELING AND OPTIMIZATION OF PIEZOELECTRIC ENERGY HARVESTING

Samia Latreche

Department of Electrical Engineering, Setif 1 University, Setif, Algeria

ABSTRACT

In this paper, the modeling, optimization and simulation results of the piezoelectric energy harvesting using bond graph approach are presented. Firstly, a lightweight equivalent model derived from the bond graph is proposed. It's a comprehensive model, which is suitable for piezoelectric seismic energy harvester investigation and power optimization. The optimal charge impedance for both the resistive load and complex load are given and analysed. Finally a bond graph approach is proposed to allow optimization of the extracted energy while keeping simplicity and standalone capability. The proposed model does not rely on any inductor and is constructed with a simple switch. The power harvested is more than twice the conventional technique one on a wide band of resistive load. The bond graph model is valid close to the analysed mode centre frequency and delivers results compared to experimental and analytical data. Furthermore, we also show that the harvester can be electrically tuned to match the excitation frequency. This makes it possible to maximize the power output for both linear and non-linear loads.

KEYWORDS

Piezoelectric Materials, Power Harvesting, Bond Graph Modeling, Mechanical Vibration, Optimization.

1. INTRODUCTION

Among various strategies for energy harvesting in micro and miso-scale, Piezoelectric Energy Harvesting System (PEHS) brought about an ideal prospect of green source of energy to power electrical devices at low driving voltage [1]. As it is expected, Micro Electro-Mechanical Systems (MEMS) and Wireless Sensor Nodes (WSN) are considered as substantial applications of PEHS in diverse fields such as biomedical, vehicles and security system.

The research community working on piezoelectric energy harvesting is a very rapidly growing community, and it includes researchers from mechanical, electrical, materials and civil engineering areas [2]. Other than the applications of piezoelectric energy harvesting, several researchers from different disciplines have studied modeling of piezoelectric energy harvesters. Certain issues have been observed in some of the existing models, and moreover, it has been noted that some very misleading models have been repeated by different researchers. The motivation here is therefore to discuss the modeling problems and provide the necessary corrections for the research community interested in bond graph modeling of piezoelectric energy harvesters.

In recent several years, researchers have proposed several methods for increasing efficiency of the piezoelectric energy harvesting systems, such as tuning proof mass [3], extending piezoelectric cantilever length by a cylindrical object [4], and choosing optimum position of piezoelectric materials on the beam cantilever [5]. However, all of these proposed approaches in the literature were solely dependent on designers' decision, which means the designers had to spend a long time in testing each possible solution to find out the final optimum dimension for a piezoelectric energy harvester in order to gain the maximum harvesting efficiency.Therefore, time-consuming design process for MEMS energy harvesters is one of the most challenging issues that the designers have to be faced with.

In this paper, an efficient method based on the bond graph model is developed and SYMBOLS (system modeling by bond graph language and simulation) software simulation for accurate estimation of performance of piezoelectric energy harvesters. According to different mechanical conditions, the system parameters used in the bond graph model are determined. This paper is organized as follows. A brief literature review is conducted in the section I. Section II describes the modeling of piezoelectric energy harvesters. A bond graph-based optimization method is constructed to optimize the dimension of piezoelectric energy harvester. Section III provides simulation results and holds some discussion on the proposed optimization methodology.

2. Analytical Modeling of Piezoelectric Harvesters

In our case, the piezoelectric harvesting is of special interest, so this transducer mechanism will be investigated in detail as follows.

2.1. Mechanical Model of Kinetic Harvesters

Inertial based kinetic energy harvesters are modelled as second-order spring-mass-damper systems. The general model of kinetic energy harvesters was first developed by Williams and Yates [1]. Figure 1 shows the general model of a kinetic energy harvester composed of lumped elements with a transducer connected to an electrical interface circuit. The framer presents the harvester which consists of a seismic mass m suspended on a spring with the stiffness k_S, generating are sonant spring-mass system.

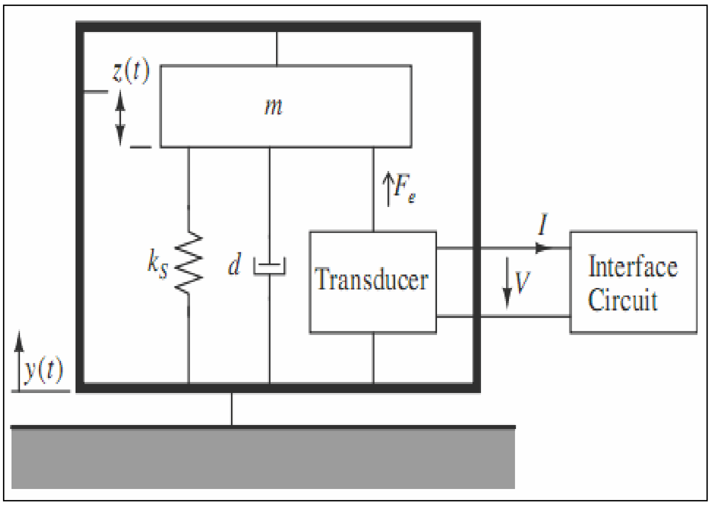


Figure 1. System of energy harvester

The spring mass damper system shown in Figure 1, can be modelled by the differential equations as indicated by Equation 1:

$$\begin{cases} ma = m(d^2 z/dt^2) + d(dz/dt) + kz + \Gamma V_p \\ I = \Gamma(dz/dt) - C_p(dV_p/dt) \end{cases} \quad (1)$$

The energy balance of the vibration harvester system can be derived as follows Equation 2 and Equation 3:

$$\underbrace{\int ma(dz/dt)\,\partial t}_{total} = \underbrace{\frac{1}{2}m((dz/dt))^2}_{kinetic} + \underbrace{\int d((dz/dt))^2\,\partial t}_{damping} + \underbrace{\frac{1}{2}kz^2}_{elastic} + \underbrace{\int \Gamma V_p(dz/dt)\,\partial t}_{electric} \quad (2)$$

$$\int \Gamma V_p(dz/dt)\partial t = \int V_p I \partial t + \frac{1}{2}C_p V_p^{\ 2} \quad (3)$$

Equation 1 states that the energy injected into the system is composed of the kinetic energy, the mechanical damping losses, the elastic energy and the energy converted into electrical energy. According to Equation 2, the energy converted into electrical energy is separated into the energy stored on the piezoelectric capacitance and the energy absorbed by the electrical load (Equation 3). The latter energy is the part which is actually being harvested.

Table 1. Notations

Parameters	**Descriptions**	**Parameters**	**Description**
z(t)	Mass displacement	y(t)	Harvester displacement
k_S	Spring stiffness	d	Damping coefficient
C_p	Piezoelectric capacitor	R_p	Piezoelectric resistance
Z	Generator impedance	R_{mc}	Matching circuit resistance
L_{mc}	Matching circuit inductance	C_{mc}	Matching circuit capacitance
(F, z)	Mechanical movement	(V, I)	Electrical movement deduced
k1	Coupling term	$V_{p,rms}$	Mean square voltage
V_{mc}	Matching circuit voltage	P_{lim}	Maximum extractable power

2.2. Equivalent Electrical System

A piezoelectric harvester can be described with an equivalent electrical system, as depicted in Figure 2.

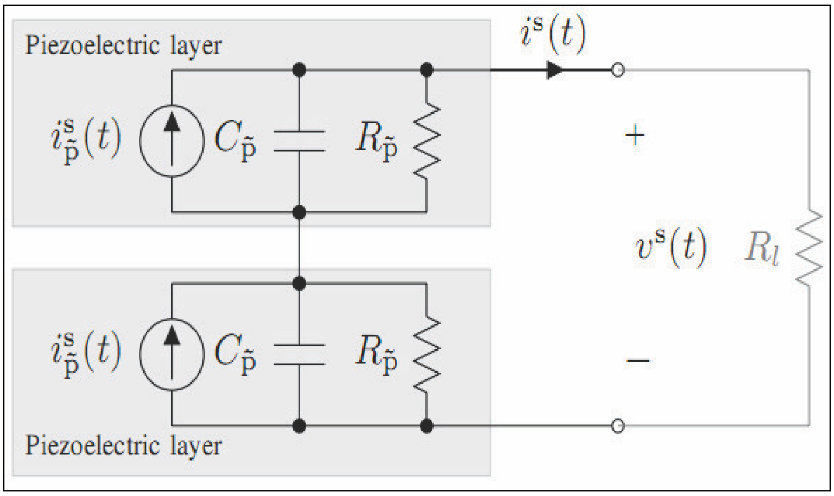


Figure 2. Equivalent circuit of piezoelectric bimorph cantilever harvester

Since the time constant of the mechanical cantilever system and internal piezoelectric inertia are far apart, a simple model of the piezoelectric element is sufficient for this analysis. More detailed models of piezoelectric transducers are available and could be of interest in future work [3, 4]. This enables the piezoelectric layer to be modelled as a simple current source, an internal capacitance and internal resistance. The electrode and substructure of the harvester create a capacitor C_p with the piezoelectric material as a dielectric. The internal resistance of the dielectric is expressed by R_p. The internal capacitance and resistance for each layer can be obtained by Equation 4.

$$C_p = \frac{\varepsilon S_{33} bL}{h_p}, R_p = Q_p \frac{h_p}{bL} \tag{4}$$

The current generated by the piezoelectric layer $i_p(t)$(Equation 5) depends on the derivative of the modal mechanical function as follows :

$$i_p(t) = \frac{k_1 \partial \eta_1(t)}{\partial t} \tag{5}$$

Where k_1: is the coupling term from the mechanical to electrical system described by Equation 6.

$$k_1 = \frac{e_{31}(h_p + h_s)b}{2} \cdot \left.\frac{\partial \Phi_1(t)}{\partial x}\right|_{x=L} \tag{6}$$

2.3. Maximum Output Power

The main figure of merit of interface circuits for any kind of generator is the power which they are able to extract. Therefore, it is helpful to know the absolute maximum power which can be extracted from the harvester. Although, this is only a theoretical value which cannot be achieved in real applications, it is never the less a means for classifying the actual extracted power. Basically, a generator with internal impedance Z outputs a maximum power if it is terminated by the load impedance equal to its conjugate complex impedance $\bar{Z}$. Using the simplified model from Figure 3, the internal impedance of a piezoelectric harvester is given as Equation 7:

$$Z = \frac{1}{jwC_p + \cfrac{1}{R_{mc} + \cfrac{1}{jwC_{mc}} + jwL_{mc}}} \tag{7}$$

By substituting the imaginary j by -j, the conjugate complex impedance can be written as Equation 8:

$$\bar{Z} = \frac{1}{-jwC_p + \cfrac{1}{R_{mc} - \cfrac{1}{jwC_{mc}} - jwL_{mc}}} \tag{8}$$

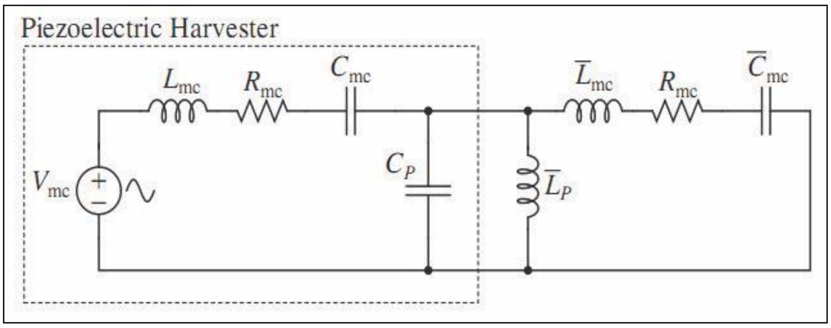


Figure 3. Piezoelectric models with conjugate complex load

Thus, in order to get an electrical equivalent circuit representing $\bar{Z}$, each capacitor within Z has to be substituted by an inductor and vice versa, leading to the equivalent circuit shown in Figure 4. The conjugate complex pair ($C_p, \bar{L}_p$) represents infinitely high impedance and the other pairs ($C, \bar{L}$) and ($\bar{L}, C$) creates zero impedance, if the values of the load impedance components are set as expressed by Equation 9:

$$\bar{L}_p = \frac{1}{w^2 C_p}, \bar{L}_{mc} = \frac{1}{w^2 C_{mc}}, \bar{C}_{mc} = \frac{1}{w^2 L_{mc}} \quad (9)$$

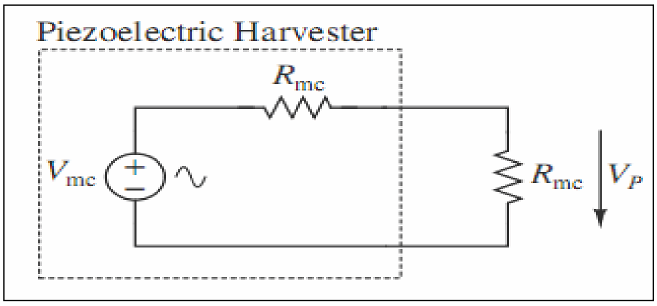


Figure 4. Simplified piezoelectric models with conjugate complex load

In this case the circuit of Figure 3 is reduced to the circuit of Figure 4, which represents a simple resistive voltage divider. The root mean square power dissipated at the load resistor represents the maximum extractable power, and it can be calculated by Equation 1:

$$P_{\lim} = \frac{V^2{}_{p,rms}}{R_{mc}} = \frac{\left(\frac{0.5\bar{\bar{V}}_{mc}}{\sqrt{2}}\right)^2}{R_{mc}} \quad (10)$$

Where $V_{p,rms}$: indicates the root mean square voltage at the load resistor. V_{mc}: denotes the amplitude of voltage source V_{mc}.

For $V_{mc} = ma/\Gamma$ and $R_{mc} = d/\Gamma^2$ as given in Equation 8 and Equation 10 can be finally expressed as shown by Equation 11:

$$P_{\lim} = \frac{m^2 a^2}{R_{mc}} \tag{11}$$

3. GRAPHICAL MODELING OF THE PIEZOELECTRIC HARVESTERS

3.1. Bond Graph Approach

Bond graph is an explicit graphical tool for capturing the structures among the physical systems and representing the mass an energy network based on the exchange of power [6]. Some authors have extended the bond graph concept to represent phenomena such as chemical kinetics and to extract causal models and control structures from the bond graph networks.

One of the advantages of bond graph method is that the models of various systems belonging to different engineering domains can be expressed using a set of only nine elements (Table 2): inertial elements (I), capacitive elements (C), resistive elements (R), effort sources (Se), flow sources (Sf), transformer elements (TF), gyrator elements (GY), 0–junctions (0-J) and 1–junctions (1-J). I, C, and R elements are passive elements that convert the supplied energy into stored or dissipated energy. Se and Sf elements are active elements supplying power to the system. TF, GY, 0-J and 1-J are junction elements that connect I, C, R, Se and Sf effectively creating the structure of the bond graph model [7 - 9].

Bond graph models are ideally suited for modeling a nonlinear system. It does not assume any linearity constraints and hides the complexity of nonlinearity from the user of the model. Once the modeller defines the nonlinear relationship in the model, it is the job of the underlying bond graph software to solve the model. The whole process is transparent to the user of the model [10, 11].

Table 2. Bond graph element and analogy between electrical and mechanical domains

Bond graph element	Electrical domain	Mechanical domain
Effort variable	Voltage	Modal coordinate
Flow variable	Current	Modal velocity
Resistive element R	Resistor	Damping
Inertial element I	Inductor	Mass
Capacitive element C	Capacitor	Spring
Effort source Se	Voltage source	Force
Flow source Sf	Current source	Speed
Transformer TF	Ideal transformer ratio	Electromechanical coupling
Gyrator GY	DC motor	Converter
0-junction (parallel connection)	Voltages equality	Forces equality
1-junction (serial connection)	Currents equality	Speeds equality

3.2. Piezoelectric Energy Harvester Optimization Methodology

Generally, the analytical models and system-level FEA are only applicable to estimate the maximum power available to be extracted from a piezoelectric energy harvester by connecting a resistive load. In practice, however, an energy harvesting circuit is required to include some nonlinear electric components such as rectifier and regulator as well as an energy storage module.

A practical energy harvesting circuit may look like the one shown in Figure 5. Furthermore, if the geometry of energy harvester is complicated so as to achieve optimal system performance, modeling of the energy harvesting system will be more challenging. Neither the graphical models nor the system-level FEA is able to address these issues. A bond graph approach is thus proposed in this work to solve these problems.

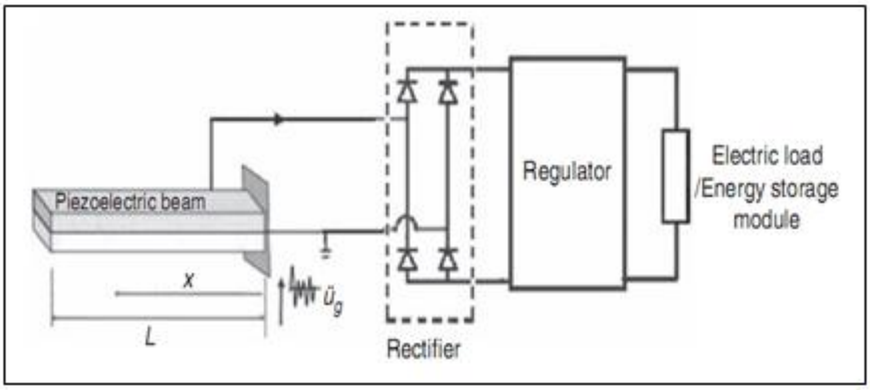


Figure 5. Practical energy harvesting circuit

To achieve an optimal output power of the cantilevered harvester, the resonant frequency should be taken into consideration. The dimensions of the cantilever and the mass decide the desirable resonant frequency of the harvester.

We consider two cases, simple and complex mechanical condition of the energy harvester. The simple mechanical condition refers to simple geometric configuration and simple mechanical boundaries. The complex mechanical condition refers to non-uniform beam configuration and complex mechanical boundaries.

SYMBOLS software is widely used in industry to simulate, verify electromechanical circuits and help the development of energy harvesting-transducers and circuits. It is highly desirable to create a model for piezoelectric vibration harvesters in bond graph and implement the model under SYMBOLS software, number of simplifications has to be made. The point of observation on the mechanical system has to be reduced to only one. The bond graph model of practical harvesting energy circuit is represented by Figure 6.

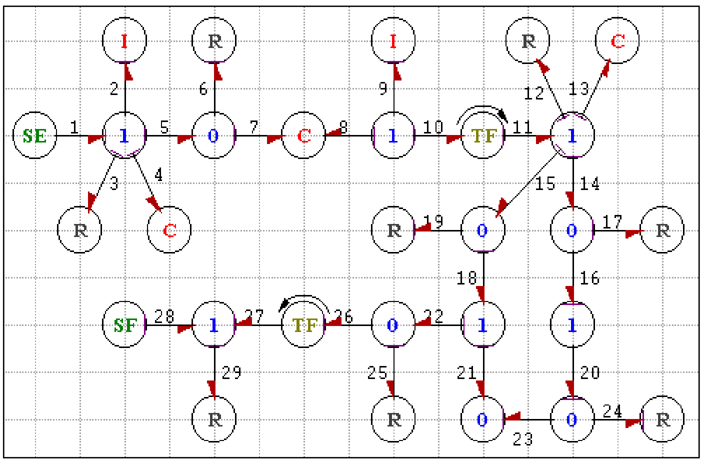


Figure 6. Bond graph model of practical harvesting energy circuit

To be able to implement a model in bond graph a number of simplifications has to be made. The point of observation on the mechanical system has to be reduced to only one point, e.g. the tip movement of the cantilever. In the models current state, only one mode can be simulate data time, therefore we selected the first mode due to its dominating part of the output and the model only considers transverse base excitation represented in Figure 7.

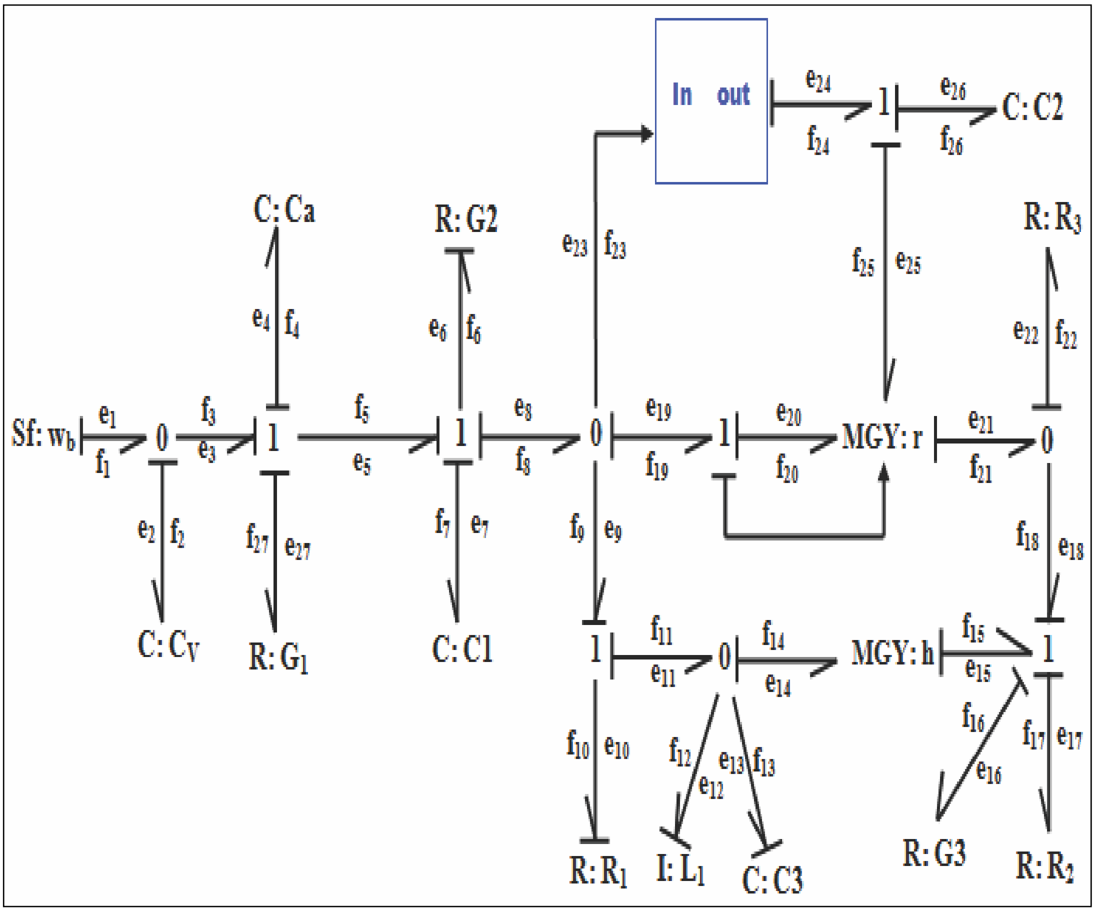


Figure 7. Bond graph model of a piezoelectric bimorph cantilever harvester

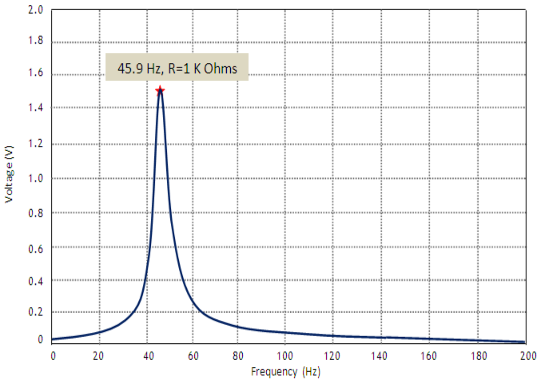
The model takes the displacement of the base as an input, where volt represents meter. The motion of the base has been reduced to transverse the displacement (Figure 7).

The displacement of input is derived in two steps to obtain the velocity and acceleration of the base. The gyrator GY converts the acceleration of the base to a force.

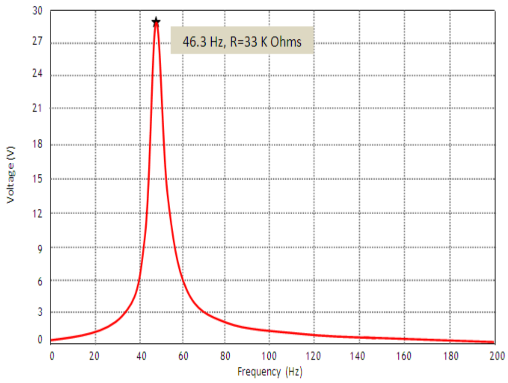
The piezoelectric layer can be modelled as a current source, internal capacitance and an internal resistance. The internal capacitance C_p and internal resistance R_p in the equivalent electrical circuit are represented by R_2, R_3, C_1 and C_2 in the bond graph model.

4. Simulation Results

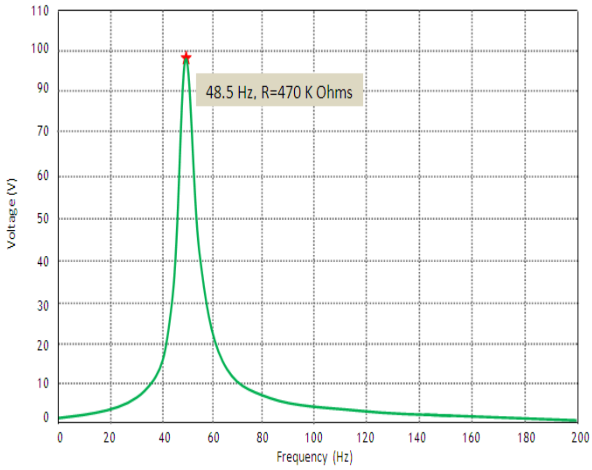
The model is validated by comparing the experimental results from [12] see Figure10. The simulation results presented below uses the same modal damping ζ=0.027 as [11], ζ which makes it possible to compare their analytical results with our simulated results.



(a) Voltage output with 1kΩ load resistor



(b) Voltage output with 33kΩ load resistor



(c) Voltage output with 470kΩ load resistor

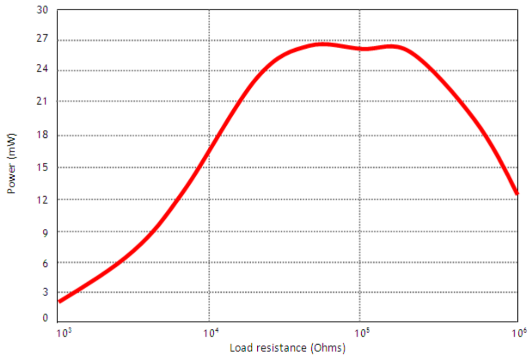
Figure 8. Simulated voltage output dependent on excitation frequency with three different load resistors

To show that the model is affected by the voltage applied/induced in the piezoelectric layers, a voltage step is applied on to the harvester and a mechanical response can be observed from the model. We also show that the model works with non-linear loads in time domain by analysing the output power from the harvester through a conventional rectifying bridge; and that the power can be optimized by controlling the voltage in the storage medium after the rectifying bridge.

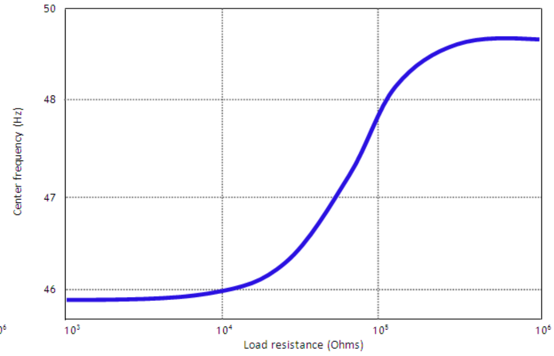
Figure 8 also shows how the resonance frequency of the harvester changes with different electrical loads. By changing the load from 1kΩ to 470kΩ, the centre frequency changes from 45.7Hz to 48.2Hz, respectively. The results of Figure 8.a, 8.b and 8.c are summarized in Table 3:

Table 3. Sensitivity of output voltage and centre frequency with linear load resistors

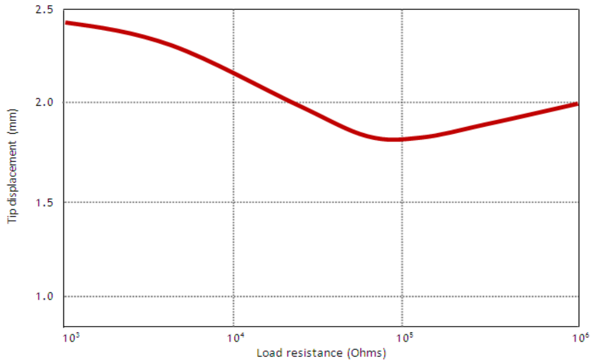
R(kΩ)	**f(Hz)**	**Vout (Volt)**
1	45.9	1.5
33	46.3	28.5
470	48.5	97.5



(a) Simulated maximum power output with linear resistive loads



(b) Excitation frequency at which maximum power can be extracted



(c)Maximum relative tip displacement with resistive loads

Figure 9. Model behavior at maximum output with respect to resistive loads

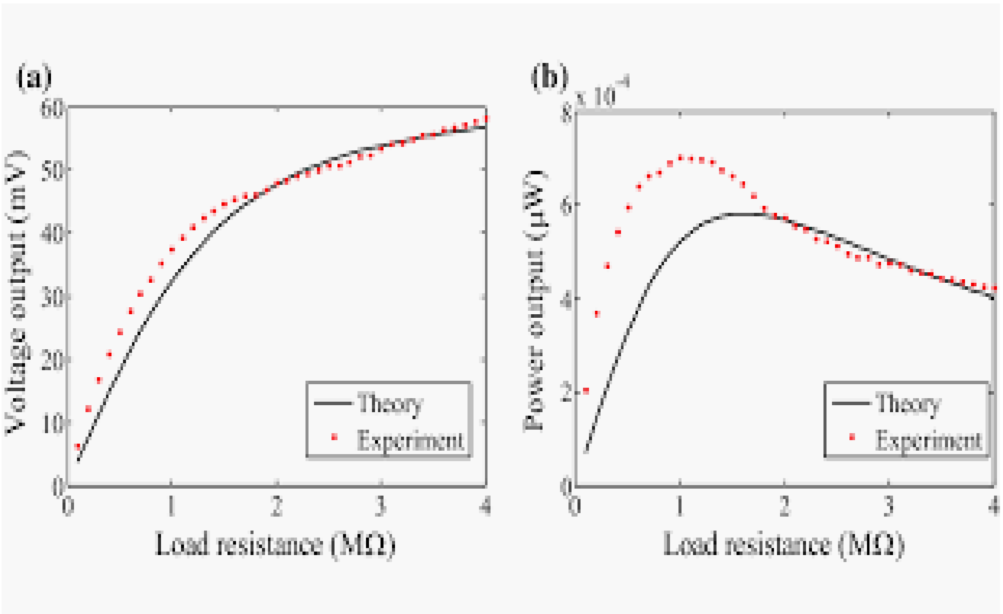


Figure 10. Linear/Nonlinear Piezoelectric Energy Harvesters from [12]

For this specific harvester, it can also be observed in Figure 9.a that the power outputs between 46Hz and 48Hz are approximately the same. The excitation frequency at which maximum power can be extracted is described by Figure 9.b. As shown in Figure 9.c, the mechanical system is heavily damped in the load region where maximum power output can be achieved [13, 14].

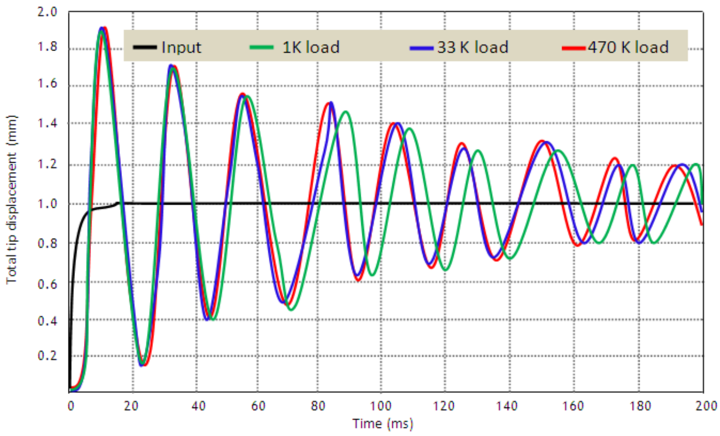


Figure 11. Tip displacement with 1mm step on the base of the harvester

By exciting the base of the harvester with a step function, the response of the system can be observed in the time domain, as shown in Figure 11.

5. CONCLUSIONS

Piezoelectric energy harvesting is a promising technique for powering small-scale standalone electronic devices.

We further proposed an effective and efficient optimization method based on bond graph for optimizing physical aspects of the piezoelectric energy harvesting systems without intensive human effort.

The work presented in this paper discussed the using of simple and reliable approach in order to optimize piezoelectric energy harvesting by focusing on modeling and circuit topology design. The bond graph model presented can take the physical dimensions and properties of a piezoelectric vibration harvester and model the behavior of it. This enables to simulate the harvester in conjunction with its harvesting circuitry and how eventual changes to the harvester affect the system.

The model also enables simulations with non-ideal electrical components, which can be used to provide a more accurate representation of a real system. The bond graph model can be used to optimize both the harvester and the harvesting circuit to fit its operating conditions.

REFERENCES

[1] N. Mohajer & M.I. Mahjoob, (2013) "Modeling and electrical optimization of a designed piezoelectric-based vibration energy harvesting system", *Proceeding of the 2013 RSI/ISM International Conference on Robotics and Mechatronics*, February 13-15, Tehran, Iran.

[2] A. Erturk & D.J. Inman, (2008) "Issues in mathematical modeling of piezoelectric energy harvesters", *Journal of Smart Materials Structures*, Vol. 17, pp. 14.

[3] Y. Jia & A.A. Seshia, (2016) "Power optimization by mass tuning for MEMS piezoelectric cantilever vibration energy harvesting", Journal of Micro-electromechanical System, Vol. 25, No. 1, pp. 108–117.

[4] X. Gao, W.H. Shih & W.Y. Shih, (2013) “Flow energy harvesting using piezoelectric cantilevers with cylindrical extension”, *Industrials Electronics IEEE Transactions*, Vol. 60, No. 3, pp. 1116–1118.

[5] H.J. Jung, Y. Song, S.K. Hong, C.H. Yang, S.J. Hwang, S.Y. Jeong & T.H. Sung, (2015) “Design and optimization of piezoelectric impact-based micro wind energy harvester for wireless sensor network”, *Sensors Actuators A Physical*, Vol. 222, pp. 314–321.

[6] H.M. Paynter, (1961) “Analysis and design of engineering systems”, *Cambridge, MA: MIT Press.*

[7] J. Thoma, (1975) “Introduction to the bond graphs and their applications”, *Pergam on Press, Oxford.*

[8] D.C. Karnopp & R.C. Rosenberg, (2009) “System Dynamics: A Unified Approach”, *John Wiley, New York*.

[9] D.C. Karnopp, D.L. Margolis & R.C. Rosenberg, (2006) “System dynamics modeling and simulation of mechatronics systems”, *New Jersey: John Wiley & Sons*.

[10] D. Champier, (2017) “Thermoelectric generators: a review of applications”, *Energy conversion and management*, Vol. 140, pp. 167-181.

[11] S. Latreche, M. Mostefai, M. Meddad, A. Eddiai, B. Sahraoui, M. Khemliche & A. Badoud, (2016) “Modeling and diagnostic of an ultrasonic piezoelectric actuator”, *Molecular Crystals and Liquid Crystals*, Vol. 628, No.1, pp. 23-40.

[12] A. Erturk & D.J. Inman, (2009) “An experimentally validated bimorph cantilever model for piezoelectric energy harvesting from base excitations”, *Smart Materials and Structures*, Vol. 18, No. 2, pp. 025009-025027.

[13] L. Guo & Q. Lu, (2017) “Modeling a new energy harvesting pavement system with experimental verification”, *Applied energy*, Vol. 208, pp. 1071-1082.

[14] J.M. Renno, M.F. Daqaq & D.J. Inman, (2009) “On the optimal energy harvesting from a vibration source”, *Journal of Sound and Vibration*, Vol. 320, No. 1-2, pp. 386 - 405